BARYON ACOUSTIC OSCILLATIONS

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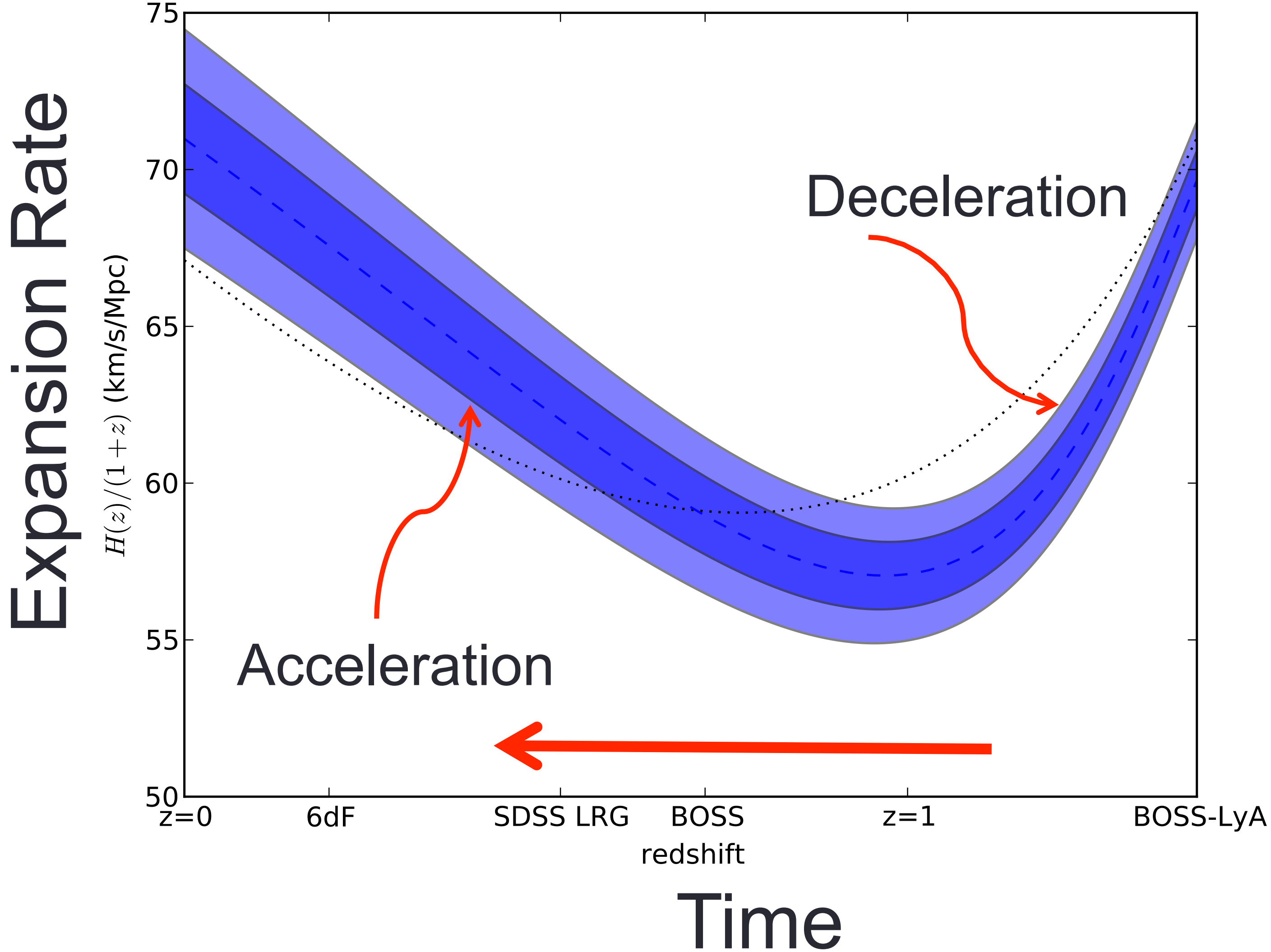
Outline

- BAO : An overview
  - Dark Energy; measuring the expansion rate
  - Standard Rulers
  - Two point statistics
  - BAO : The cartoon
  - A summary of observations
- Measuring the BAO feature
  - Parametrizing cosmology
  - The anisotropic BAO feature
  - More fitting details
- The nonlinear BAO feature
- Open questions, future surveys

The Questions in Cosmology

- We have a rich tapestry of cosmological observations that point to a standard model of cosmology.
- Three threads :
  - Inflation
  - Dark Matter
  - Dark Energy
- The questions in cosmology are changing
  - No longer understanding how these give rise to the Universe we live in.
  - Understanding the physics underlying these three.

The expanding Universe



Dark Energy : The Big Questions

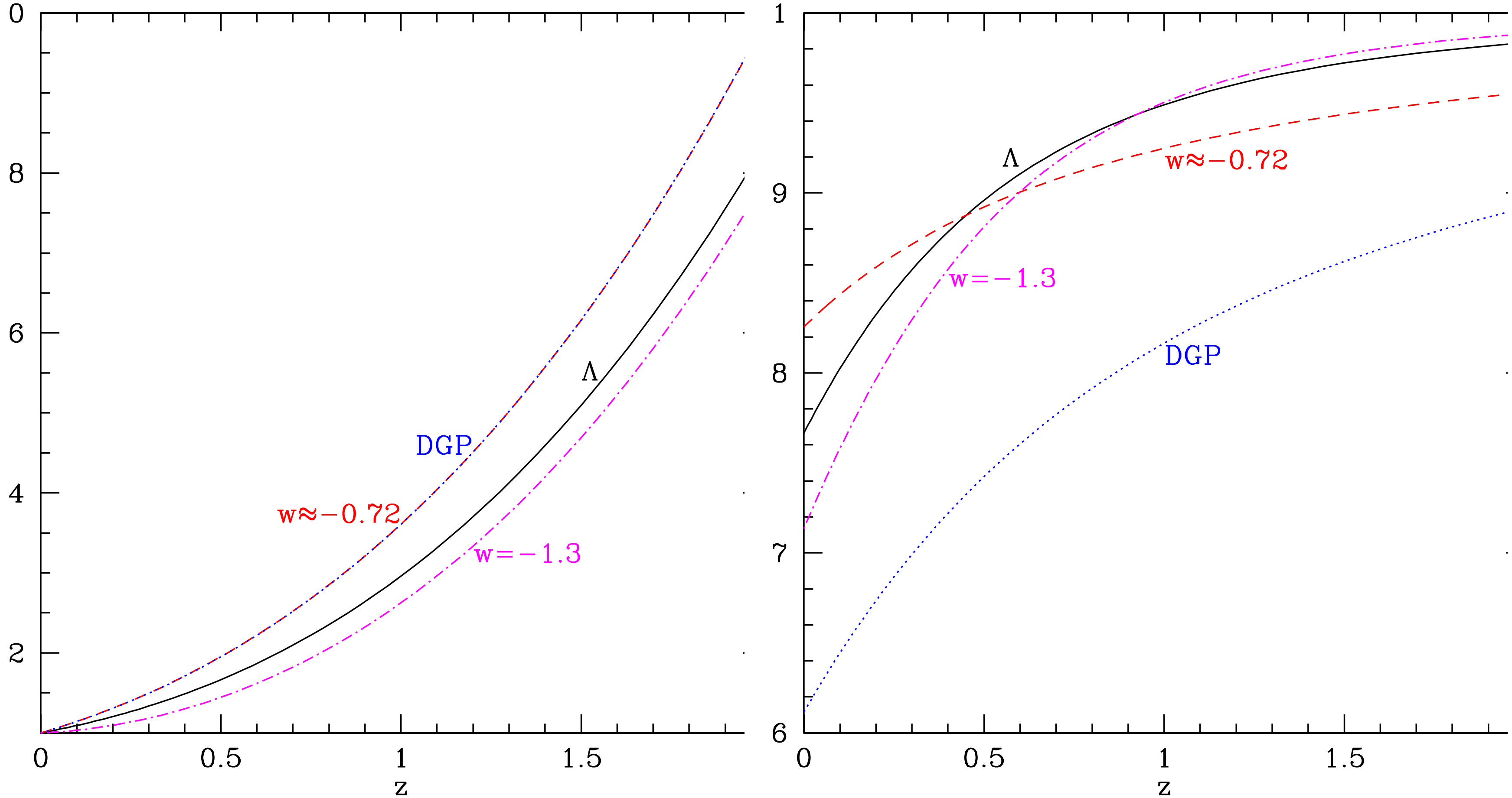
- Is cosmic expansion accelerating because of a breakdown of GR on cosmological scales or because of a new energy component that exerts repulsive gravity within GR?
- If the latter, is it consistent with a cosmological constant or does it evolve in time?
- Any answers to this will point to new physics!

Dark Energy

- Measure the expansion rate of the Universe
  - The distance-redshift relations
  - Directly measure H(z)
- Measure the rate at which structures grow in the Universe
  - Growth function D(z), and its derivatives
- Two paradigms
  - Dark Energy
    - What is its equation of state? How does it evolve with redshift?
    - Is it consistent with a cosmological constant?
  - Modified gravity
    - How do structures form in the Universe?
    - Are matter and light affected the same way?
- This is a rich set of questions, and requires multiple probes.

From Eric Linder

Dark Energy vs Modified Gravity



Expansion and growth probe different aspects

Probes of Dark Energy

	Expansion	**+Growth**
Imaging	Supernovae	Lensing / Clusters
Spectro	**BAO**	Redshift Distortions

These are somewhat artificial distinctions

Information/Robustness from combining different probes

Cosmology 101

Expansion rate of the Universe

$$H^2(a) = H_0^2 \left[\Omega_R a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_{DE} \exp\left\{ 3 \int_a^1 \frac{da'}{a'} [1 + w(a')] \right\} \right].$$

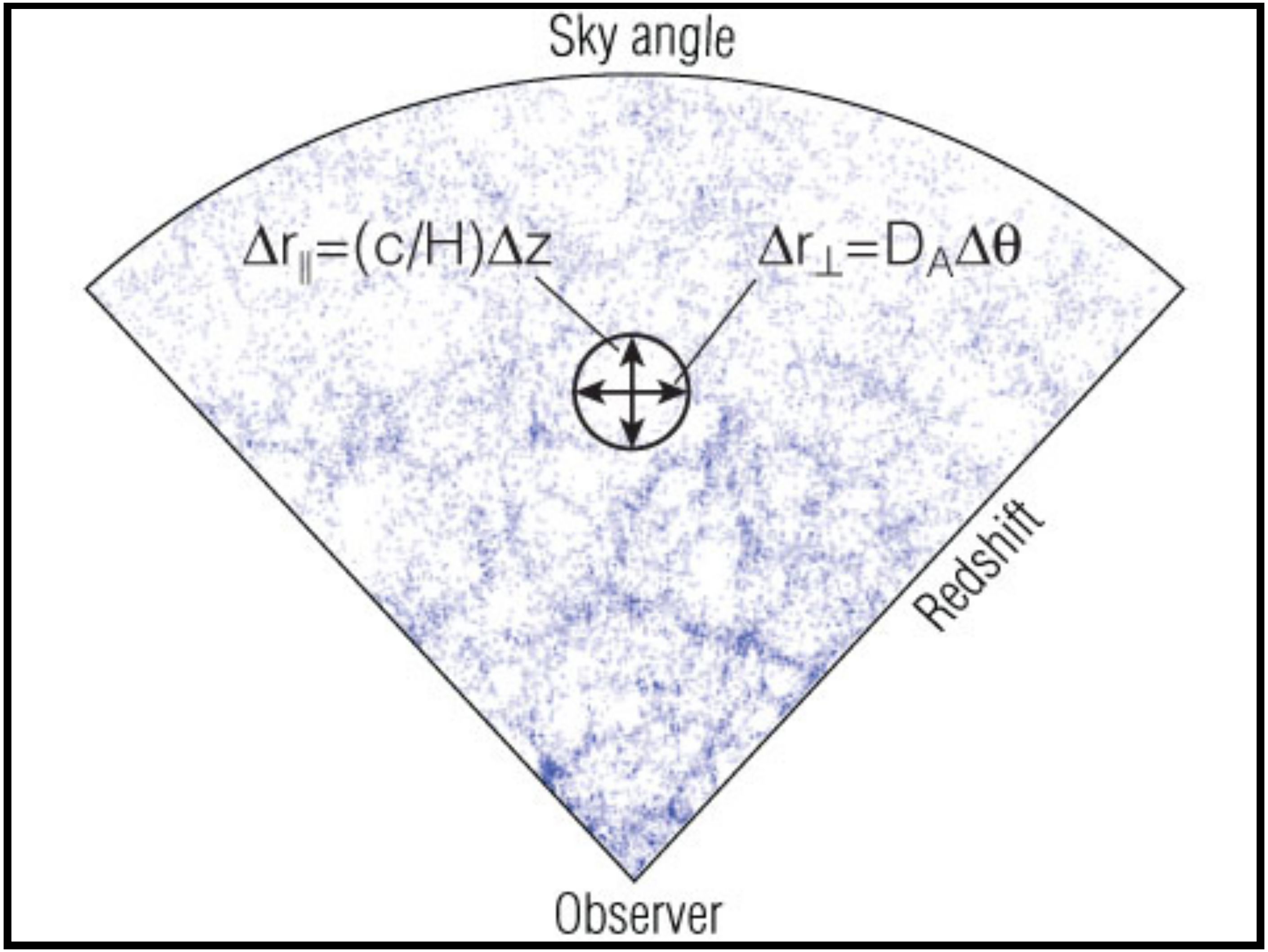
$$w(a) = w_0 + w_a(1 - a),$$

Distances (Comoving, and angular diameter)

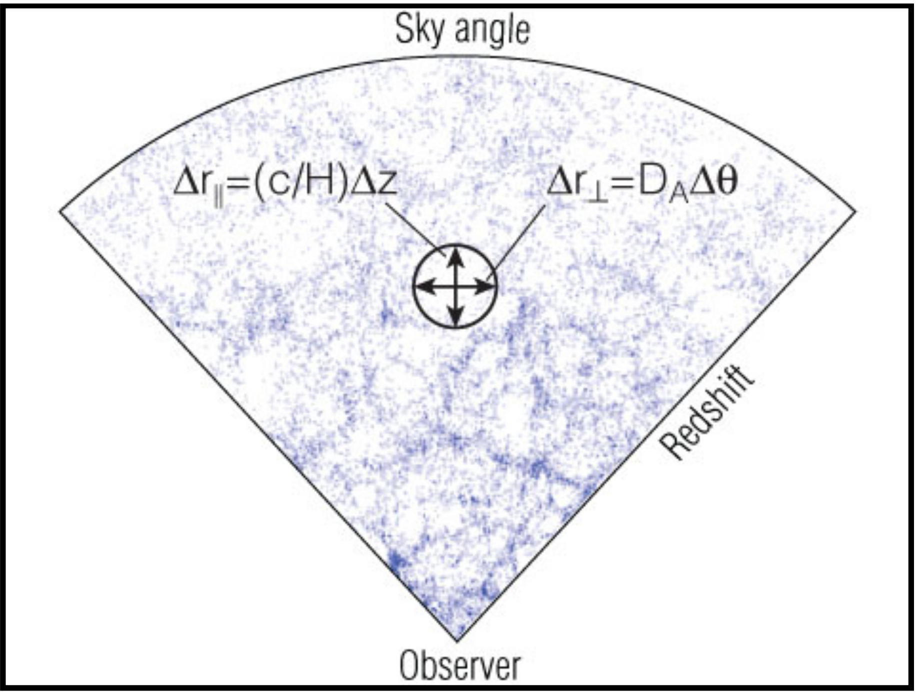
$$D_C(z) = \frac{c}{H_0} \int_0^z dz' \frac{H_0}{H(z')} .$$

$$D_A(z) = K^{-1/2} \sin\left(K^{1/2} D_C \right)$$

Measuring two distances with standard rulers



Measuring $d_A(z)$ and H(z)



- Transverse scale measures angular diameter distance
- Radial scale measures the Hubble constant
- Internal consistency tests
- H(z) unique amongst dark energy probes
- H(z) important to constrain dark energy at high redshifts

Two point statistics

- Characterize the density field by its two-point correlations
  - For a Gaussian random field, this is a complete description of the entire density field
- For BAO, useful to think both in configuration space (correlation functions) and Fourier space (power spectra)
- “Just” FTs of one another

Notation

$$
\begin{aligned}
\delta(\mathbf{x}) &= \frac{\rho(\mathbf{x})-\bar{\rho}}{\bar{\rho}} = \frac{\delta\rho}{\rho}(\mathbf{x}) \\
\delta(\mathbf{k}) &= \int d^3x\ \delta(\mathbf{x})\, e^{i\mathbf{k}\cdot\mathbf{x}} \\
\langle\delta(\mathbf{k})\delta^\star(\mathbf{k}')\rangle &= (2\pi)^3\delta_D(\mathbf{k}-\mathbf{k}')P(k) \\
\Delta^2(k) &= \frac{k^3P(k)}{2\pi^2} \\
\xi(x) &= \int \frac{d^3k}{(2\pi)^3}P(k)e^{i\mathbf{k}\cdot\mathbf{x}} \\
&= \int \frac{dk}{k}\ \Delta^2(k)j_0(kr)
\end{aligned}
$$

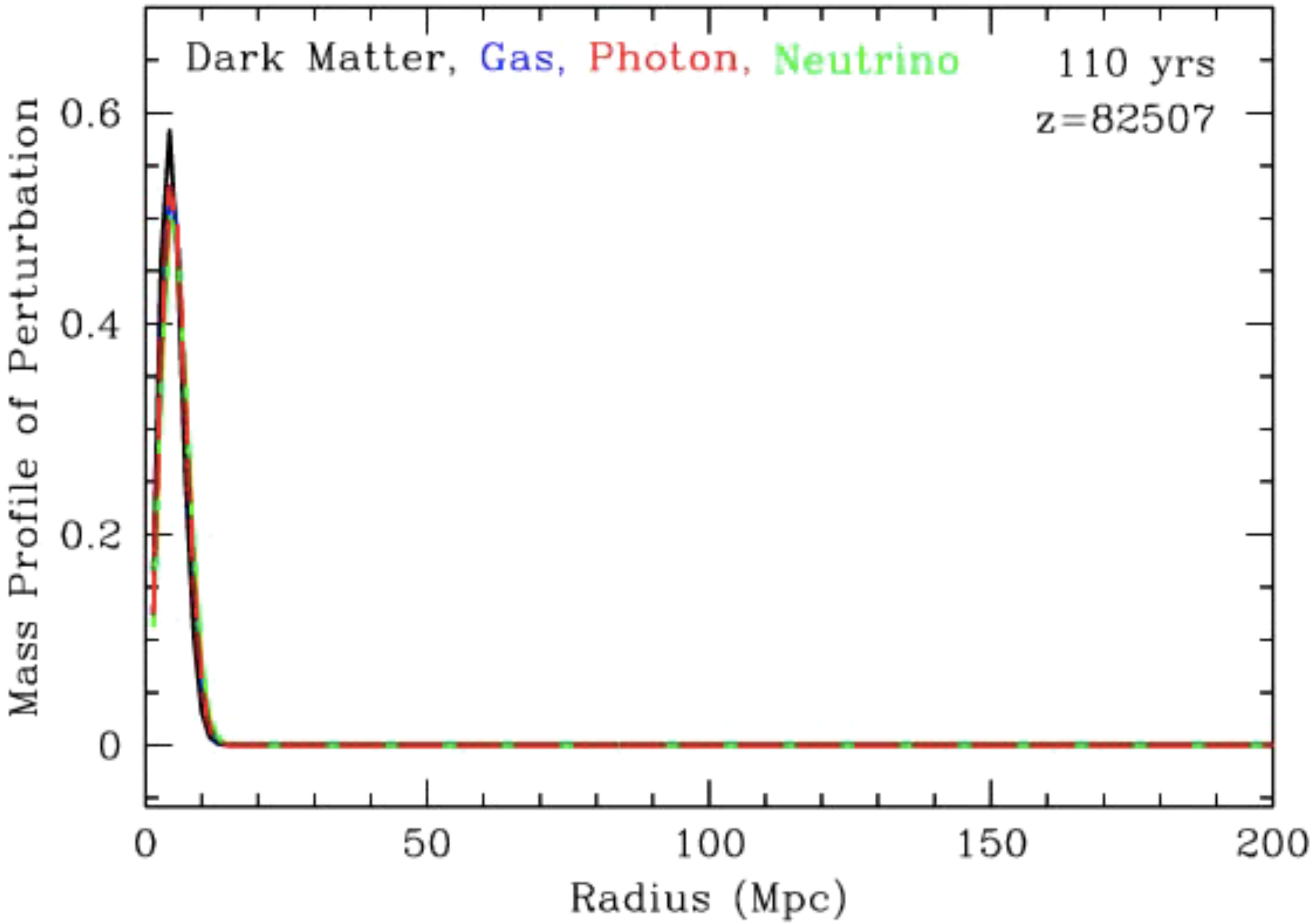
Borrowed from M. White

Constructing a Standard Ruler

- The plasma of the early Universe supports sound waves
  - Compton scattering between electrons and photons
  - Coulomb interactions between electrons and protons
- Sound waves from the initial density perturbations expand outward
  - Speed of sound ~ c/√3
- When the Universe cools below 0.3 eV, electrons and protons "recombine"
  - Sound wave stalls, leaving imprint on density fluctuations.
  - Characteristic scale of 153.2 Mpc ~ 4.7e24 m

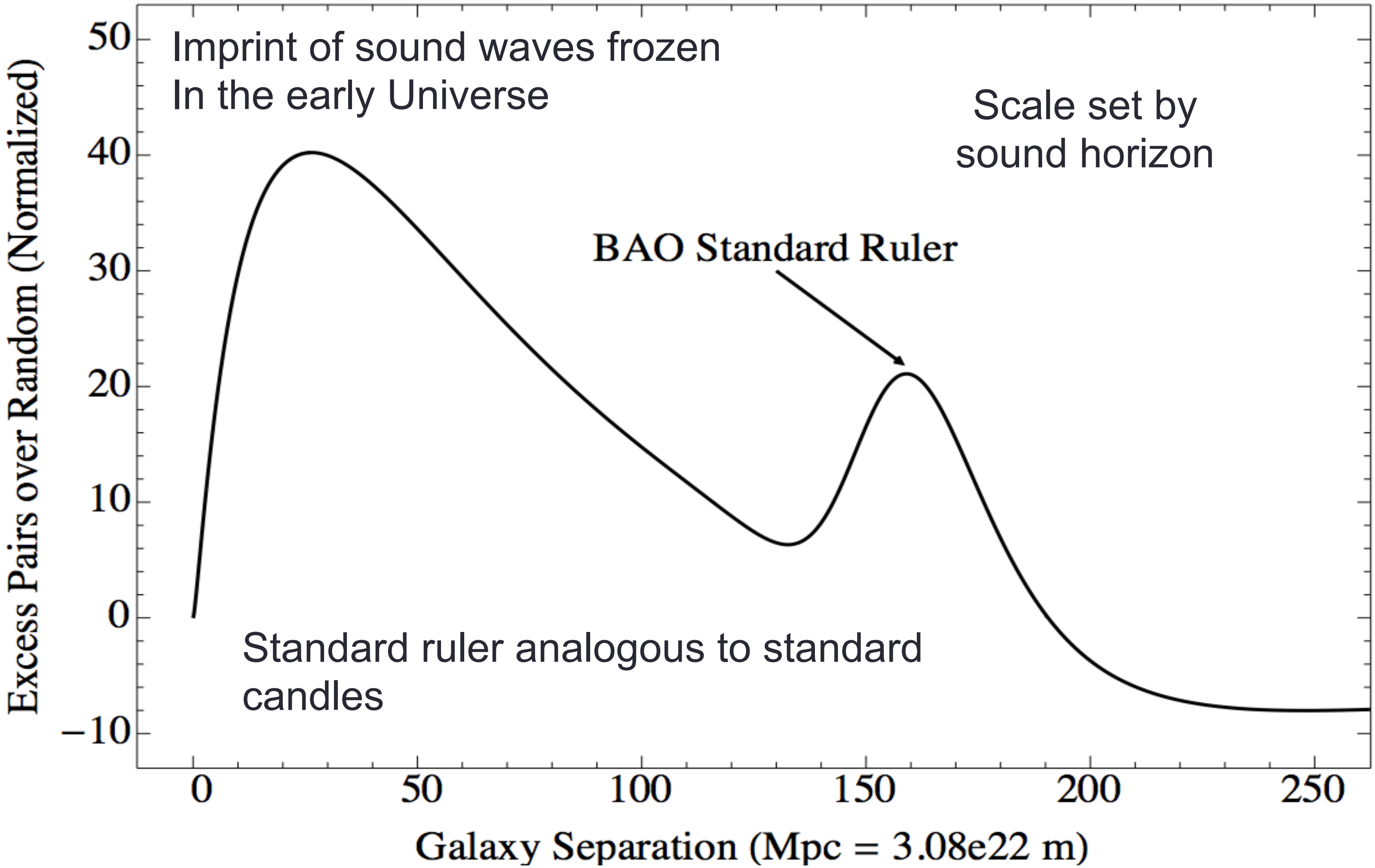
$$s = \int c_s(1+z)\,dt = \int \frac{c_s\,dz}{H(z)} = \frac{1}{\sqrt{\Omega_m H_0^2}} \frac{2c}{\sqrt{3 z_{\rm eq} R_{\rm eq}}} \ln \frac{\sqrt{1+R_{\rm dec}} + \sqrt{R_{\rm dec}+R_{\rm eq}}}{1+\sqrt{R_{\rm eq}}}$$

Sound Waves imprint a Standard Ruler



Daniel Eisenstein

The Standard Ruler in the Galaxy Correlation Function



Constructing a BAO survey

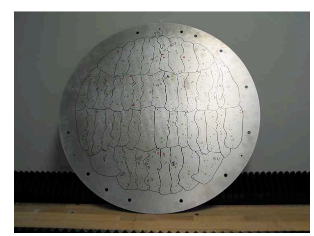
Survey

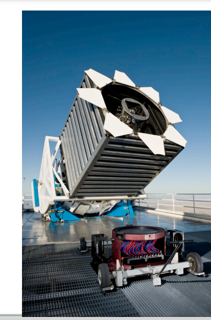
Image Sky

SDSS-I/II imaged 14K sq. deg (1/3 of sky)
~ billion objects detected

1.5M

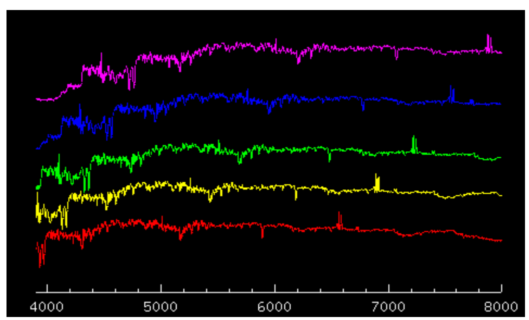
Select objects



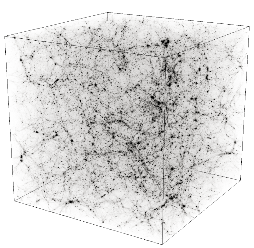


Get Spectra

1000 at a time



Measure redshifts



3D Map

Constructing a BAO survey

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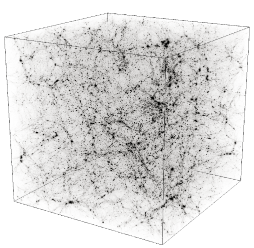

3D Map

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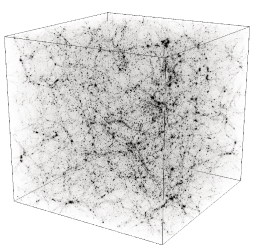
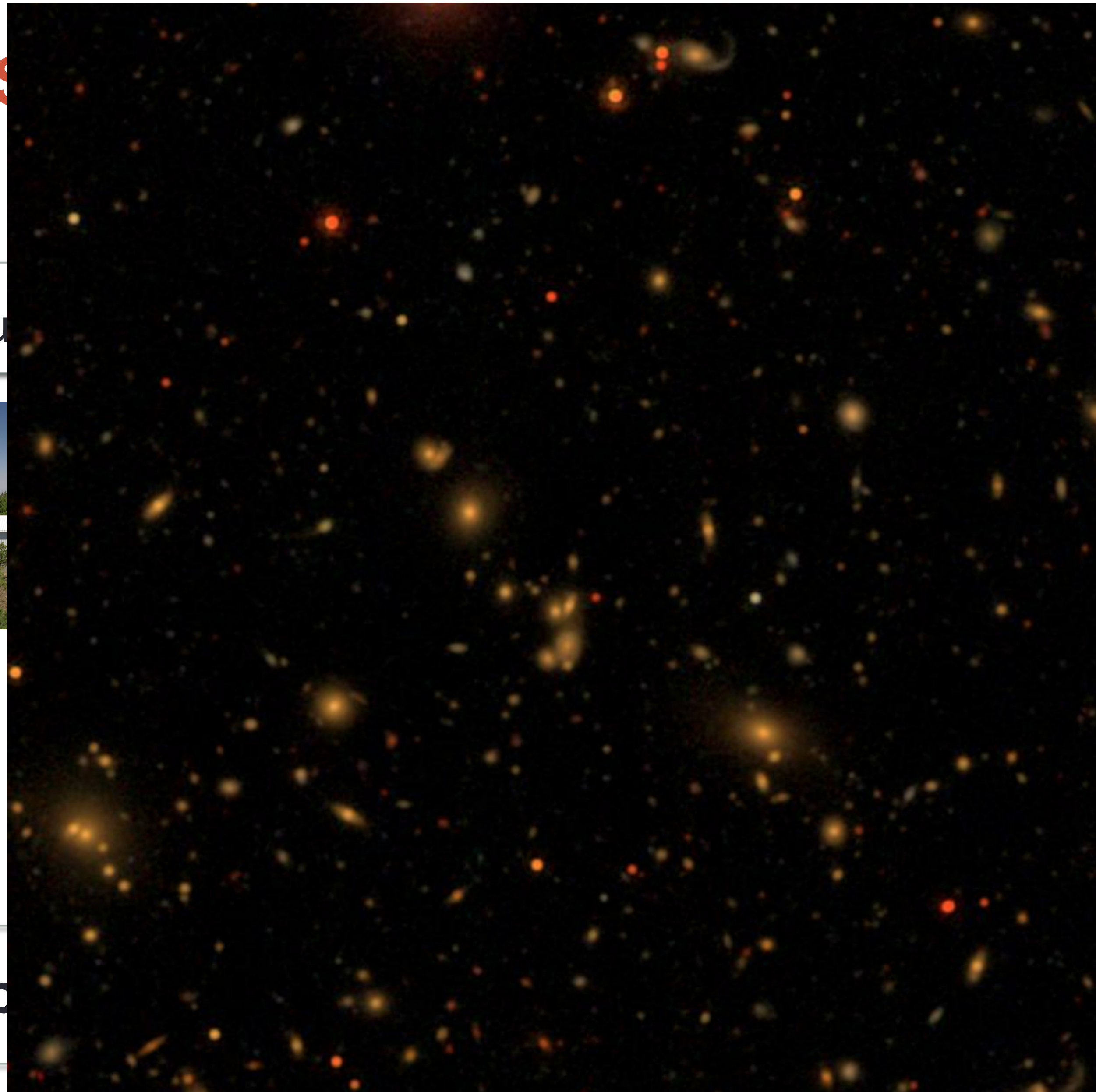


3D Map



/II imaged 14K
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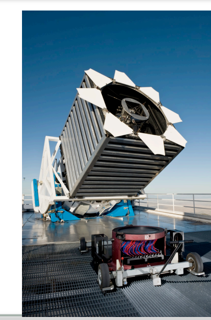

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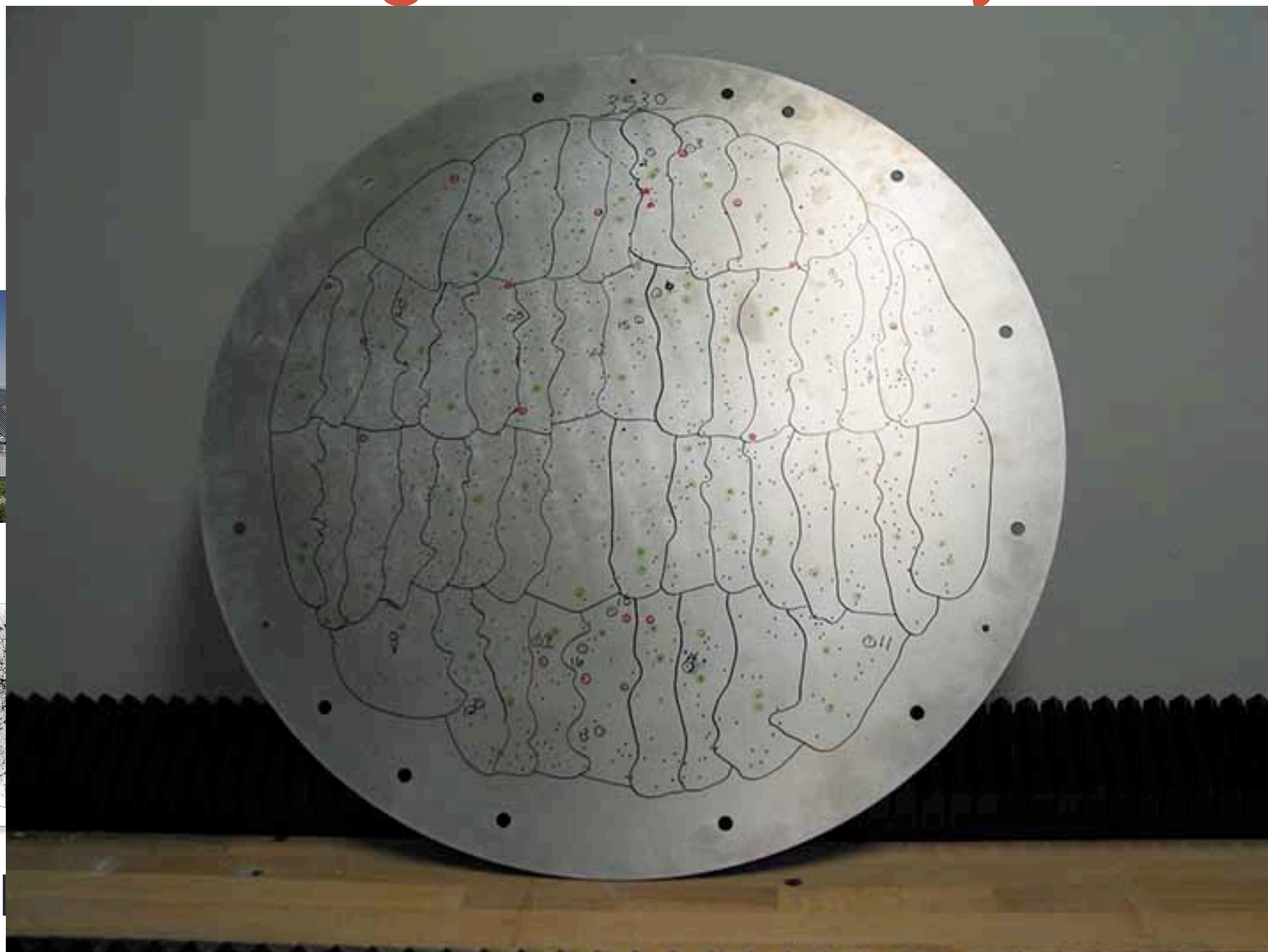
Constructing a BAO survey



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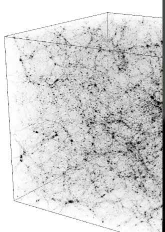

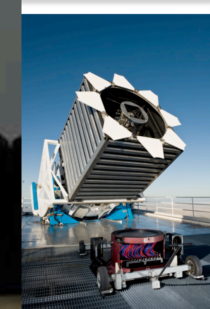

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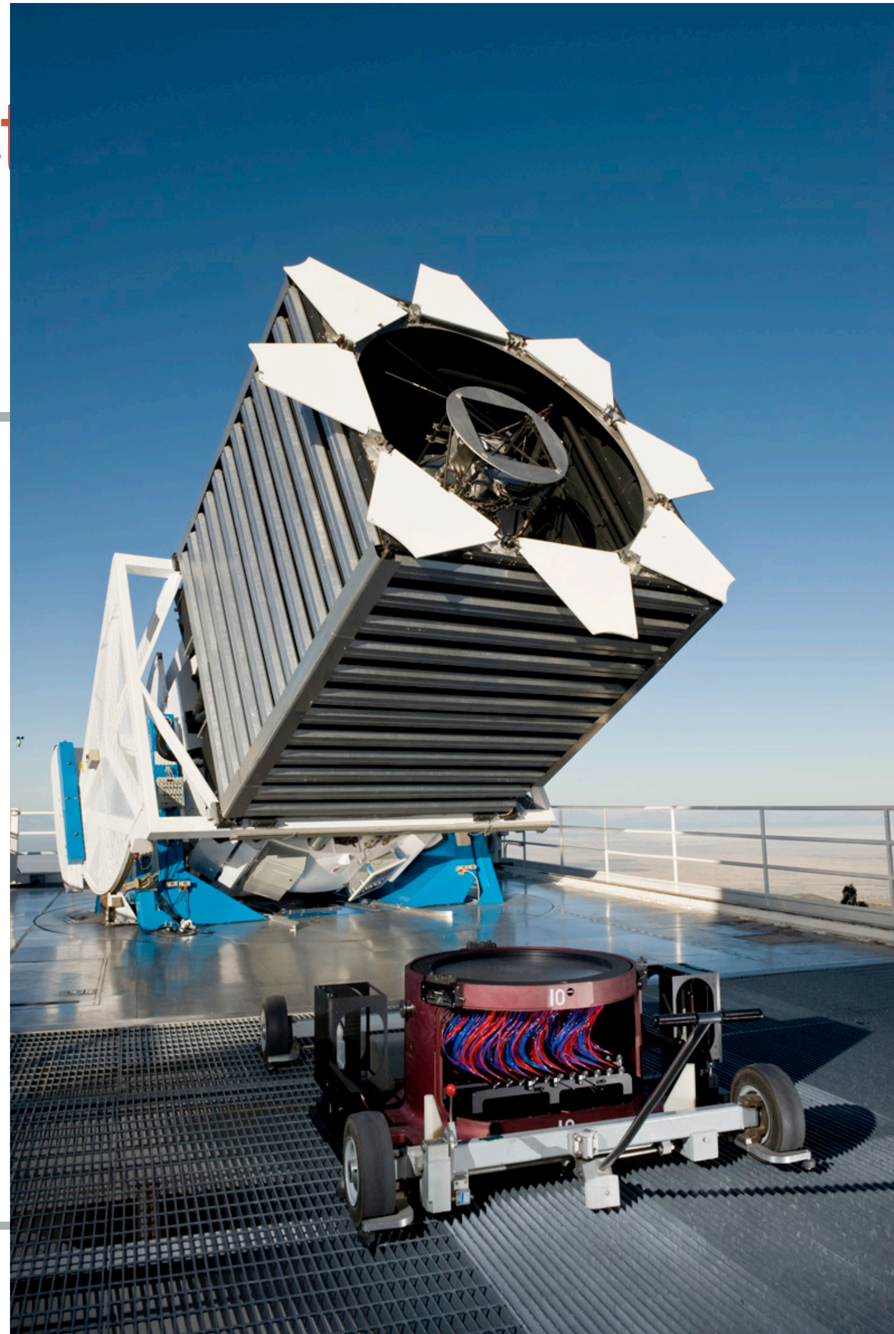
3D

1000 at a time

Construct



Survey

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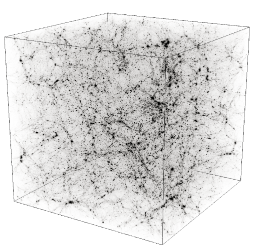
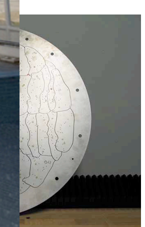
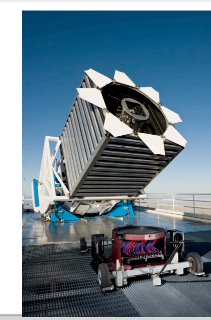
Select objects





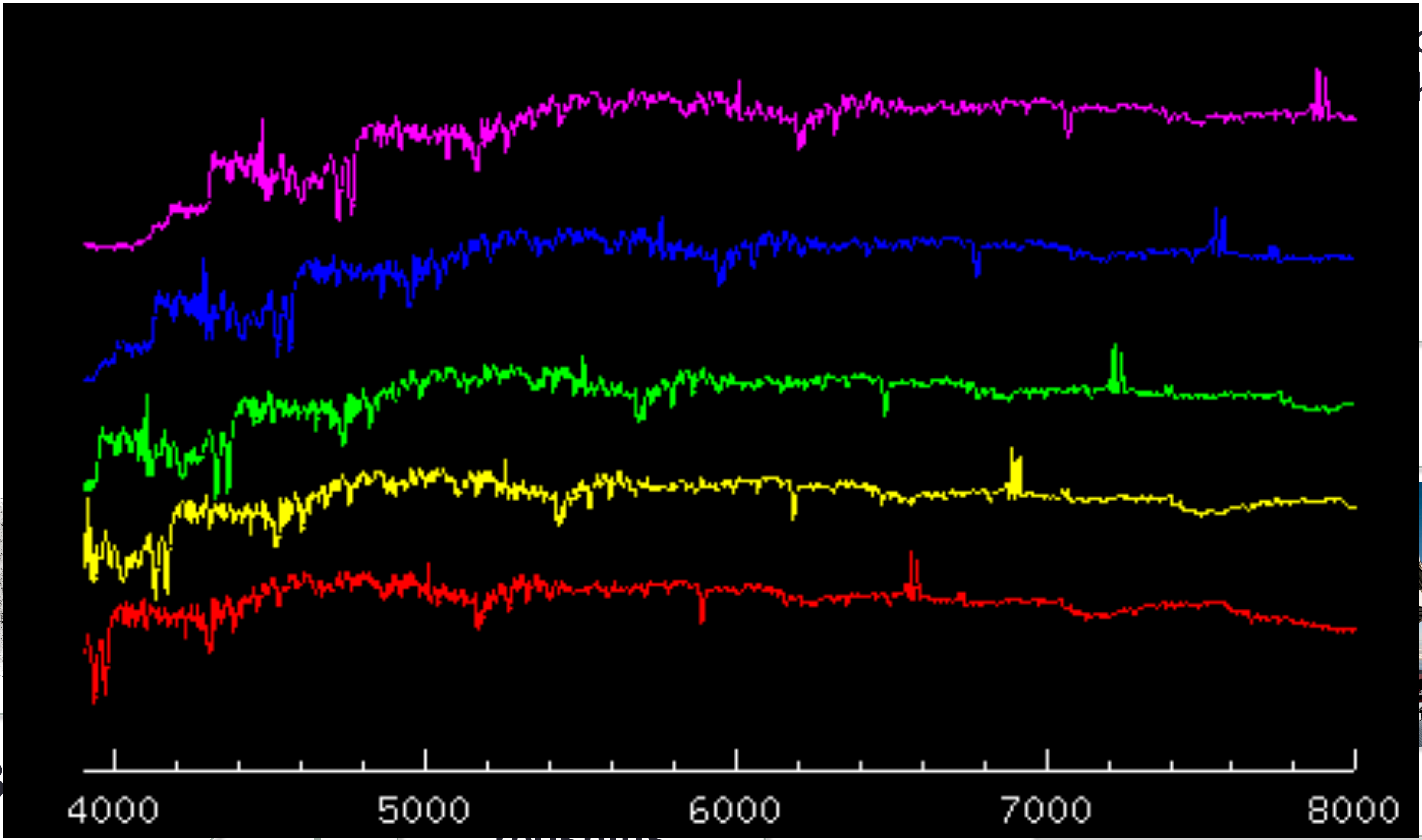


3D Map

Get Spectra

1000 at a time

Constructing a BAO survey



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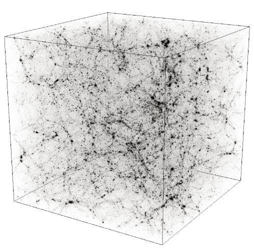
redshifts

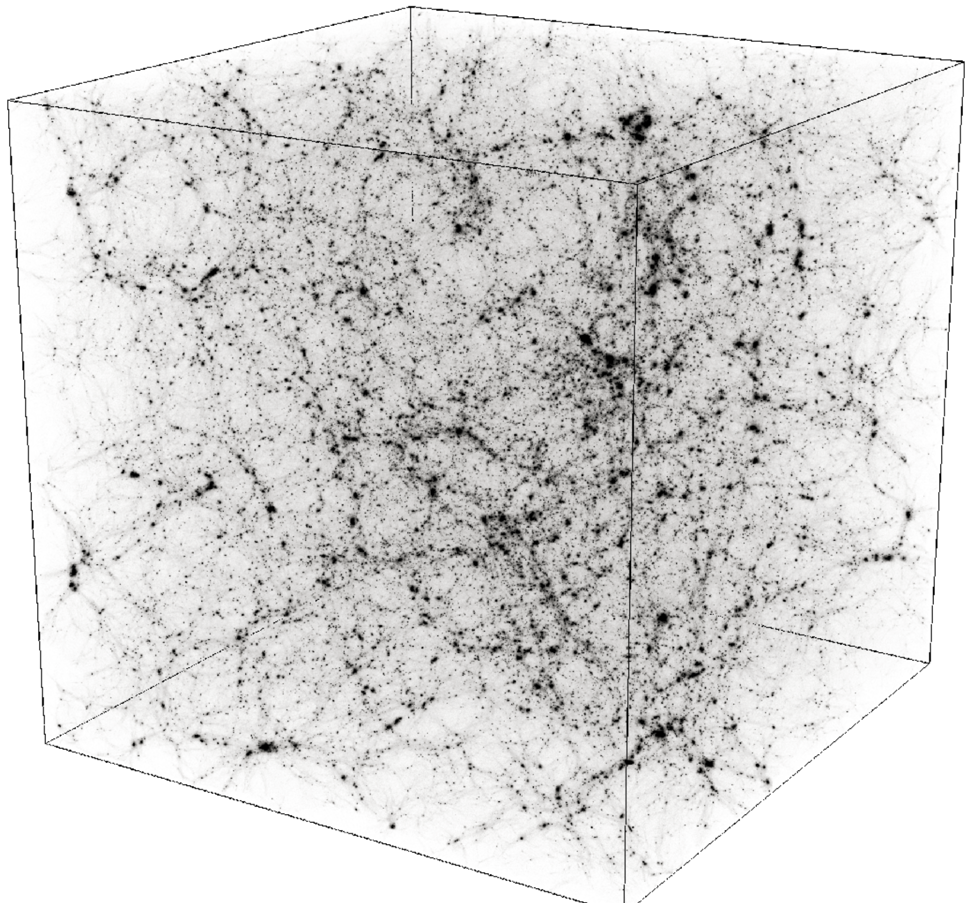
1000 at a time

Constructing a BAO survey

Surve



3D Map



S-I/II imaged 14K
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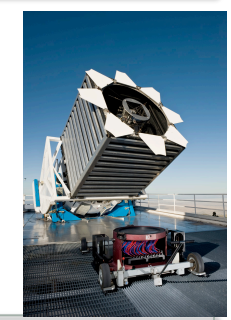
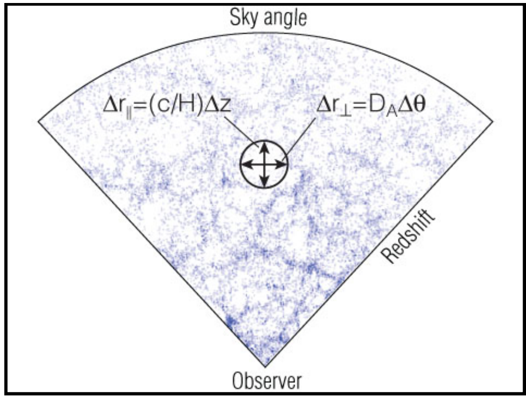


Spectra

1000 at a time

Observables

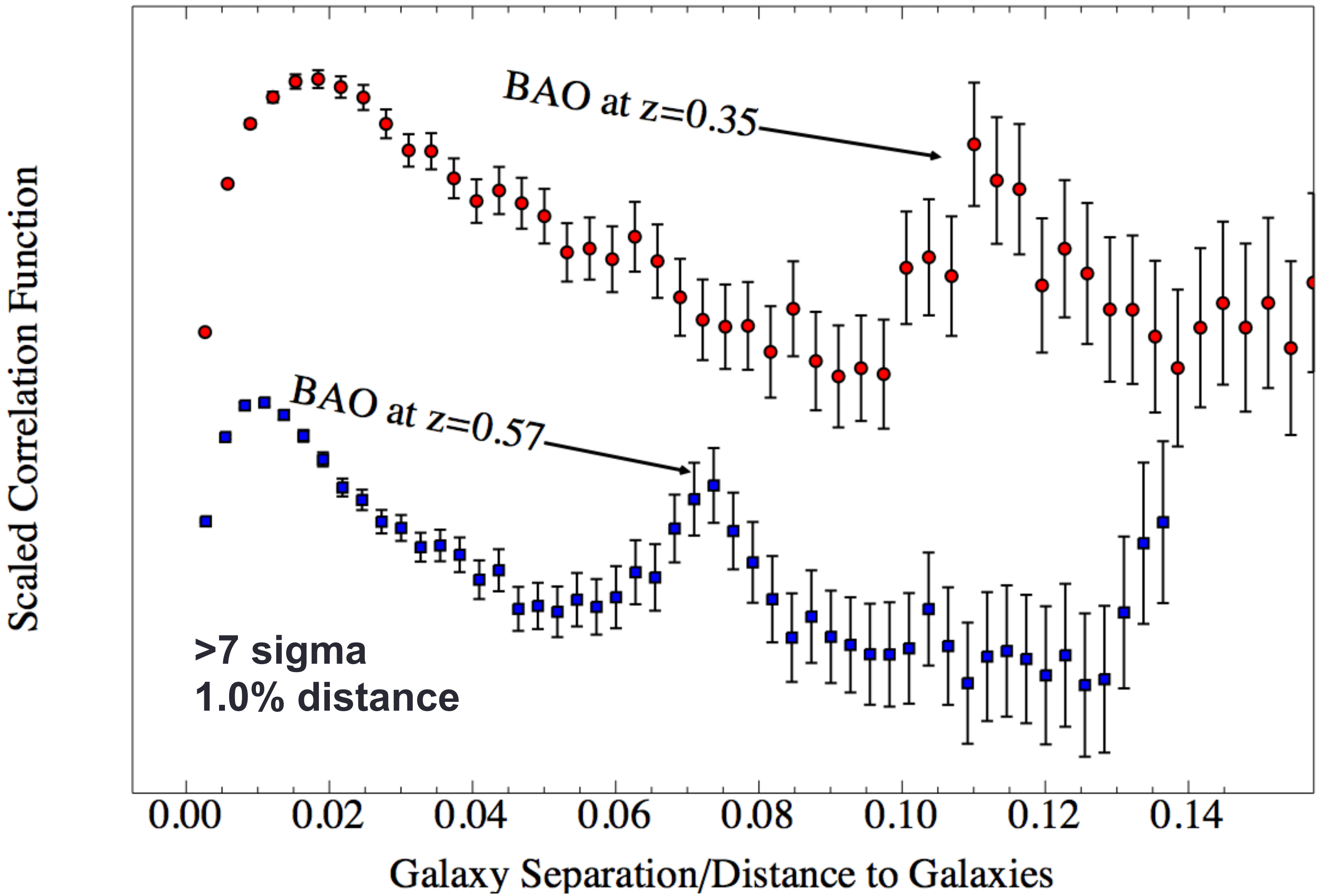
- Positions on the sky and redshifts
  - 3D map of the Universe
  - Precision redshifts require a spectroscopic survey
- Need to convert angular separations to physical distances
  - Ruler oriented transverse to line of sight measures distance to the ruler.
  - Distance as a function of redshift
  - Integrated expansion rate
- Need to convert redshift separations to physical distances
  - Ruler oriented parallel to the line of sight measures rate of change of distance with redshift.
  - Expansion rate.
- Not possible with standard candles.



Why BAO?

- Simple measurement
  - Only requires positions
- Underlying theory is simple
  - Mostly linear physics (fluctuations are 1 part in 10^4)
  - Exquisitely calibrated by the CMB (~1% with WMAP, much better with Planck)
  - 3D feature (hard to mimic)
  - Very large scales >> scales of astrophysical complications
    - Can be treated perturbatively

BOSS measures the BAO standard ruler



BAO Experiments : Past, Present

Survey	Redshift	Years	Precision
2dFGRS	0.2	Completed	detection
SDSS-I/II	**0.35**	**Completed**	**2%**
WiggleZ	0.7	Completed	4%
BOSS	**0.35, 0.55, 2.5**	**2009-2014**	**1.0% at z=0.55 today**
BOSS	0.35, 0.55, 2.5	2009-2014	1% (0.35, 0.55), 1.5% (2.5)
HETDEX	3.0	2013-2015	1%

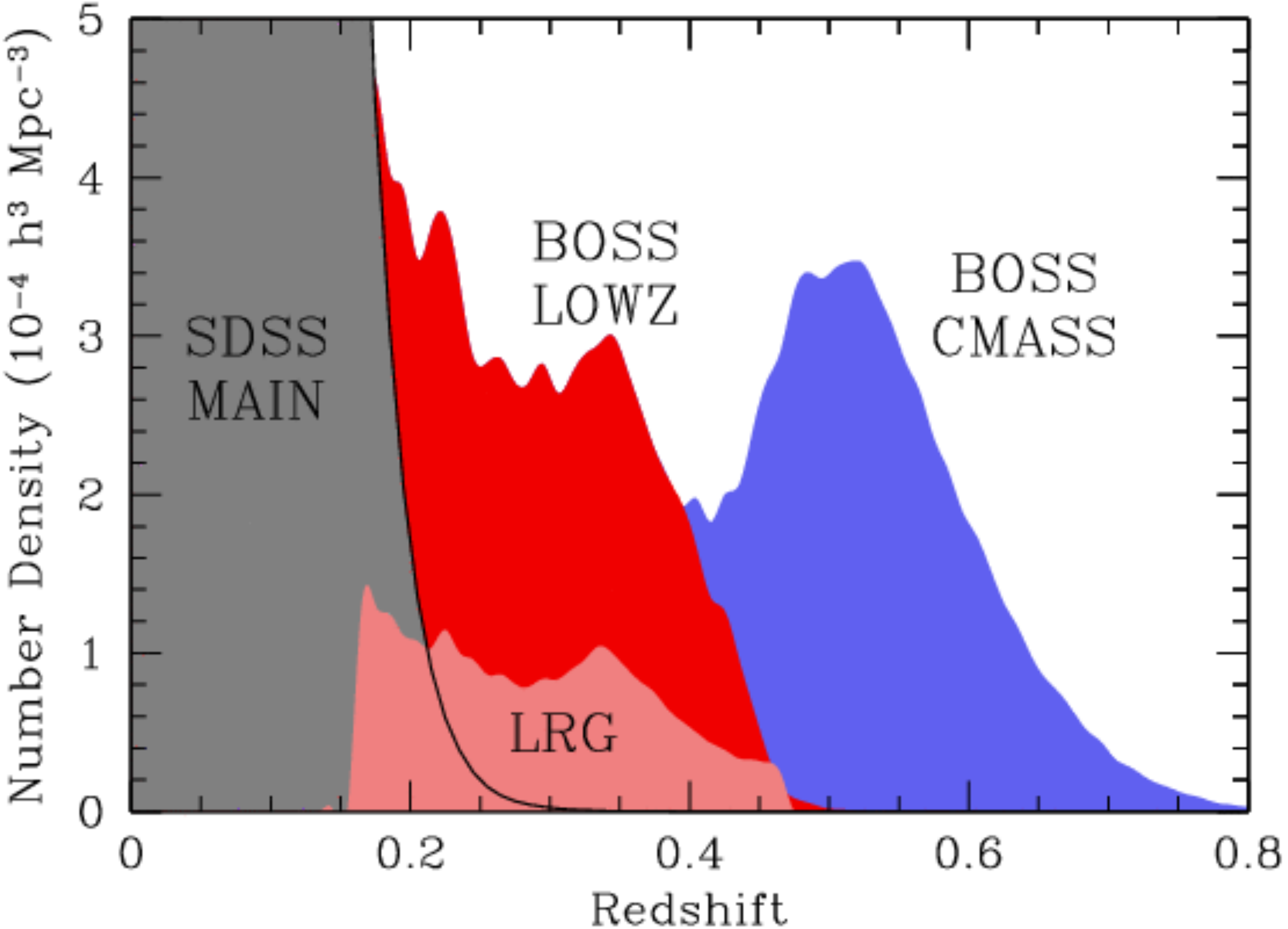
What is BOSS?

- Baryon Oscillation Spectroscopic Survey
- BAO with galaxies, Lyman-alpha forest
- On going dark energy experiment
  - Funded by DoE, NSF, Sloan Foundation and Participating Institutions
- The definitive low redshift BAO measurement
- 1% distance measurements at z=0.35, 0.6
- First results with the almost-complete sample!

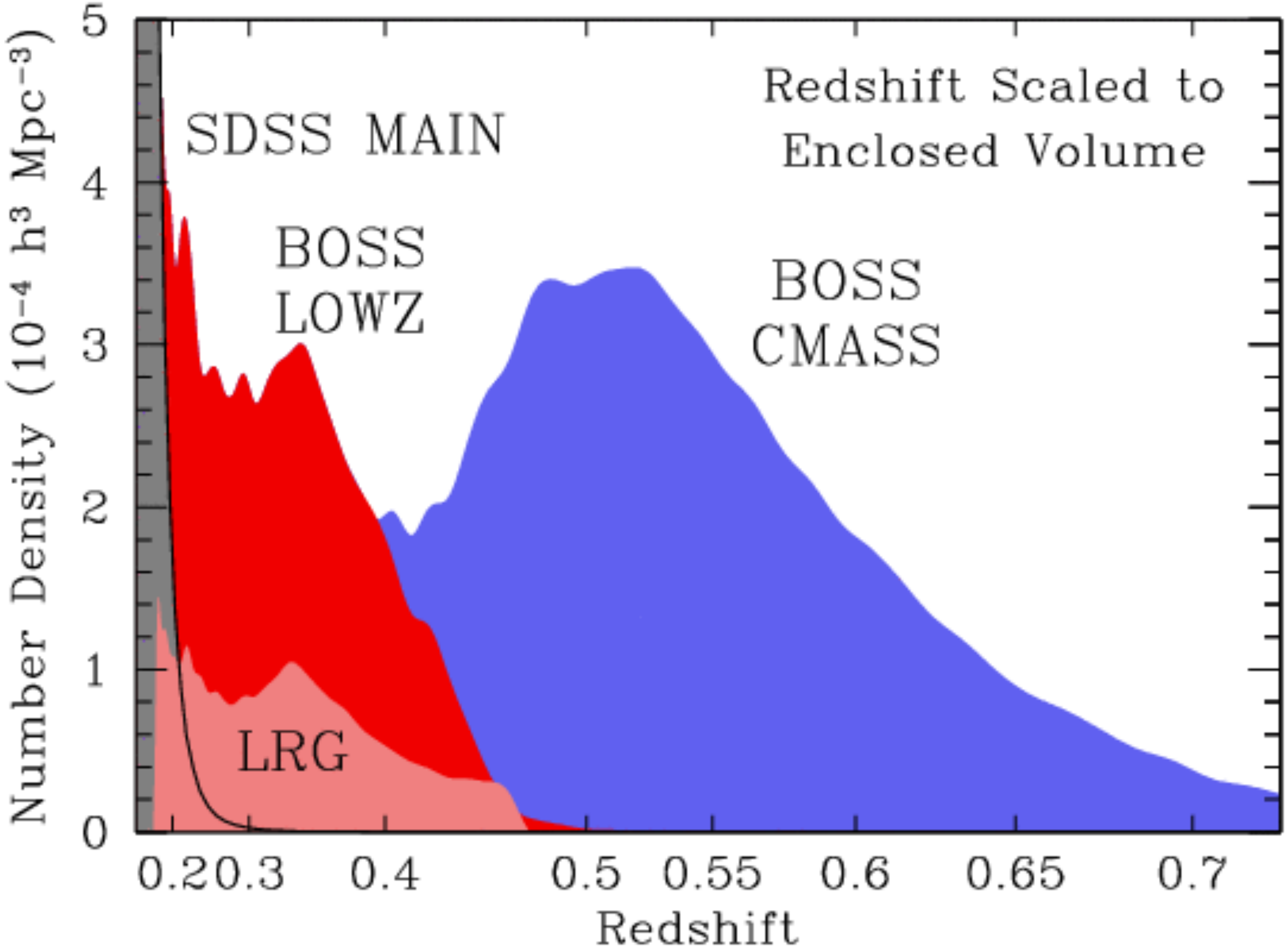
A BOSS Factsheet

	BOSS
Telescope	SDSS 2.5m
Number of simultaneous spectra	1000
Survey duration	2009-2014
Total number of galaxies	1.5 million
Distance precision from galaxy BAO	1% at z=0.35, z=0.6

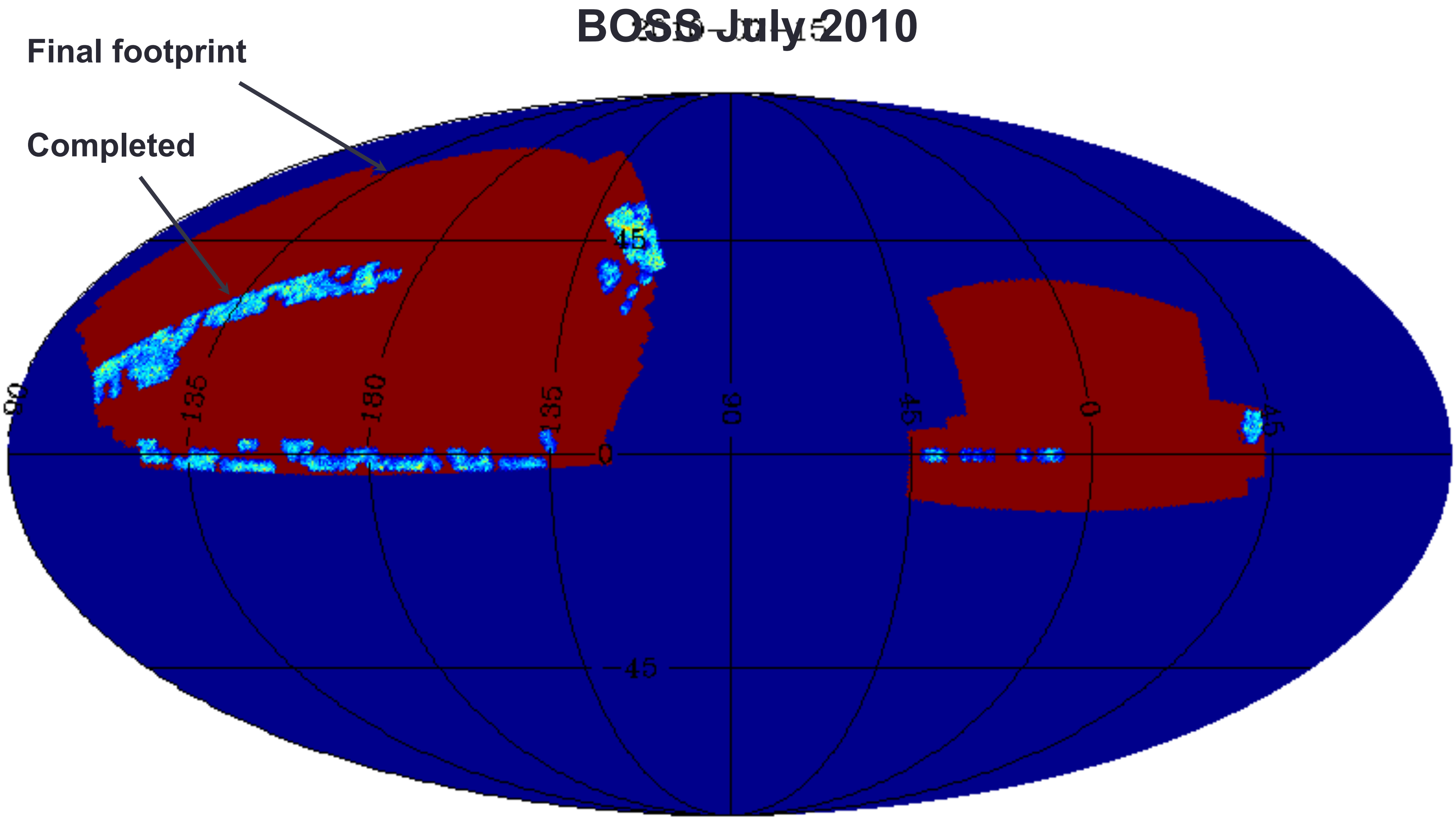
BOSS pushes out to higher redshift



… and surveys a larger volume



BOSS survey progress

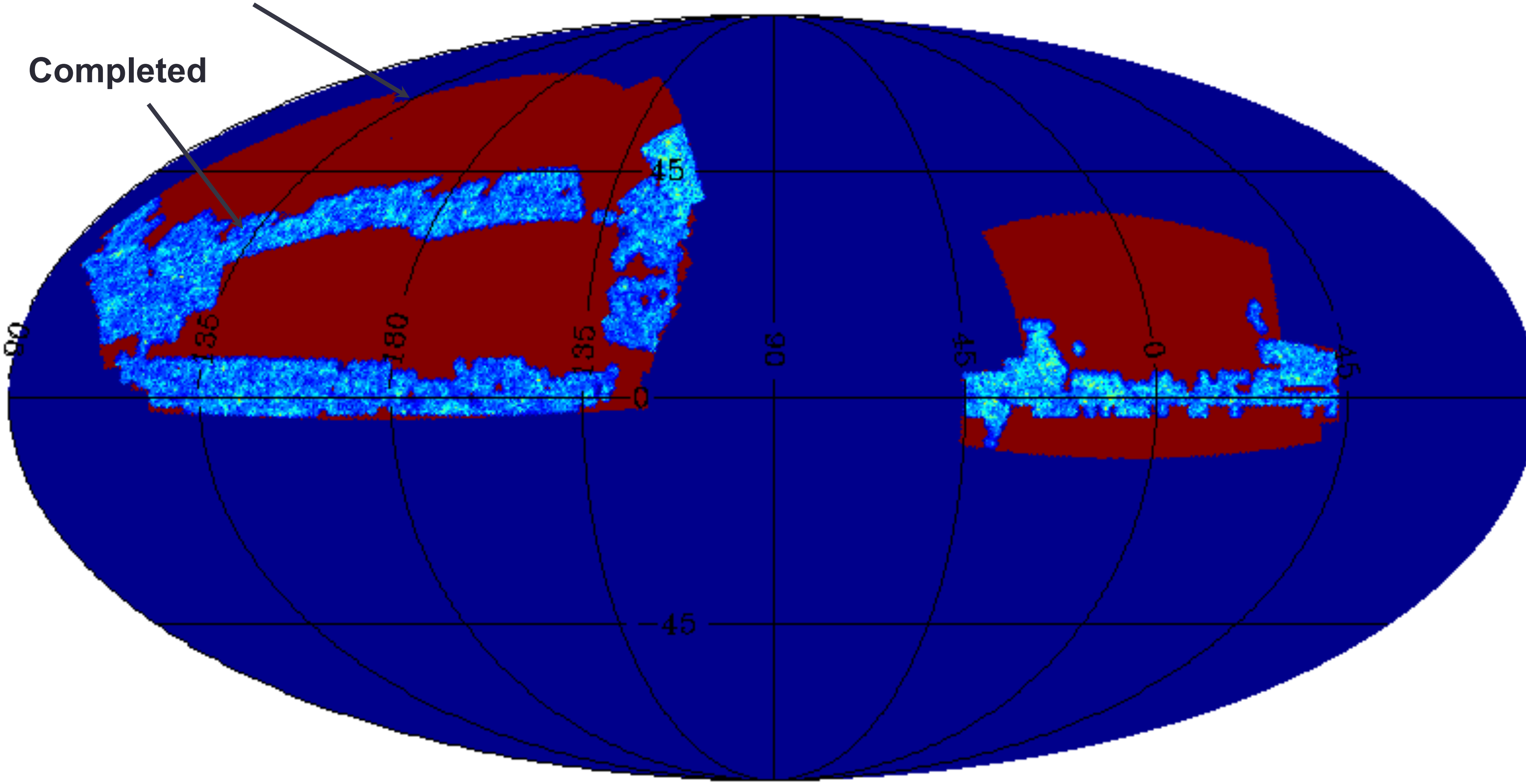


BOSS survey progress

BOSS July 2011 (Data Release 9)

Final footprint

Completed

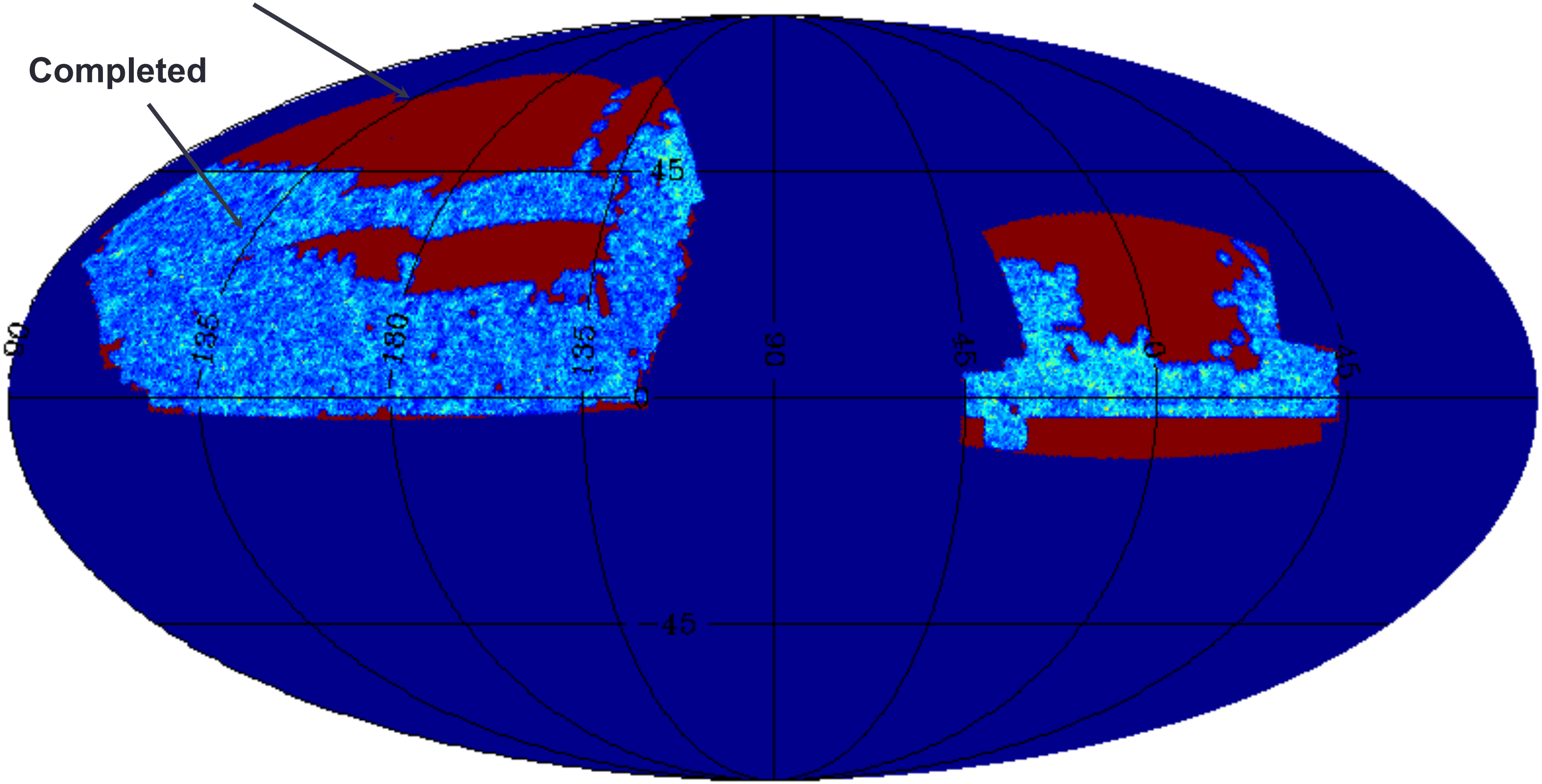


BOSS survey progress

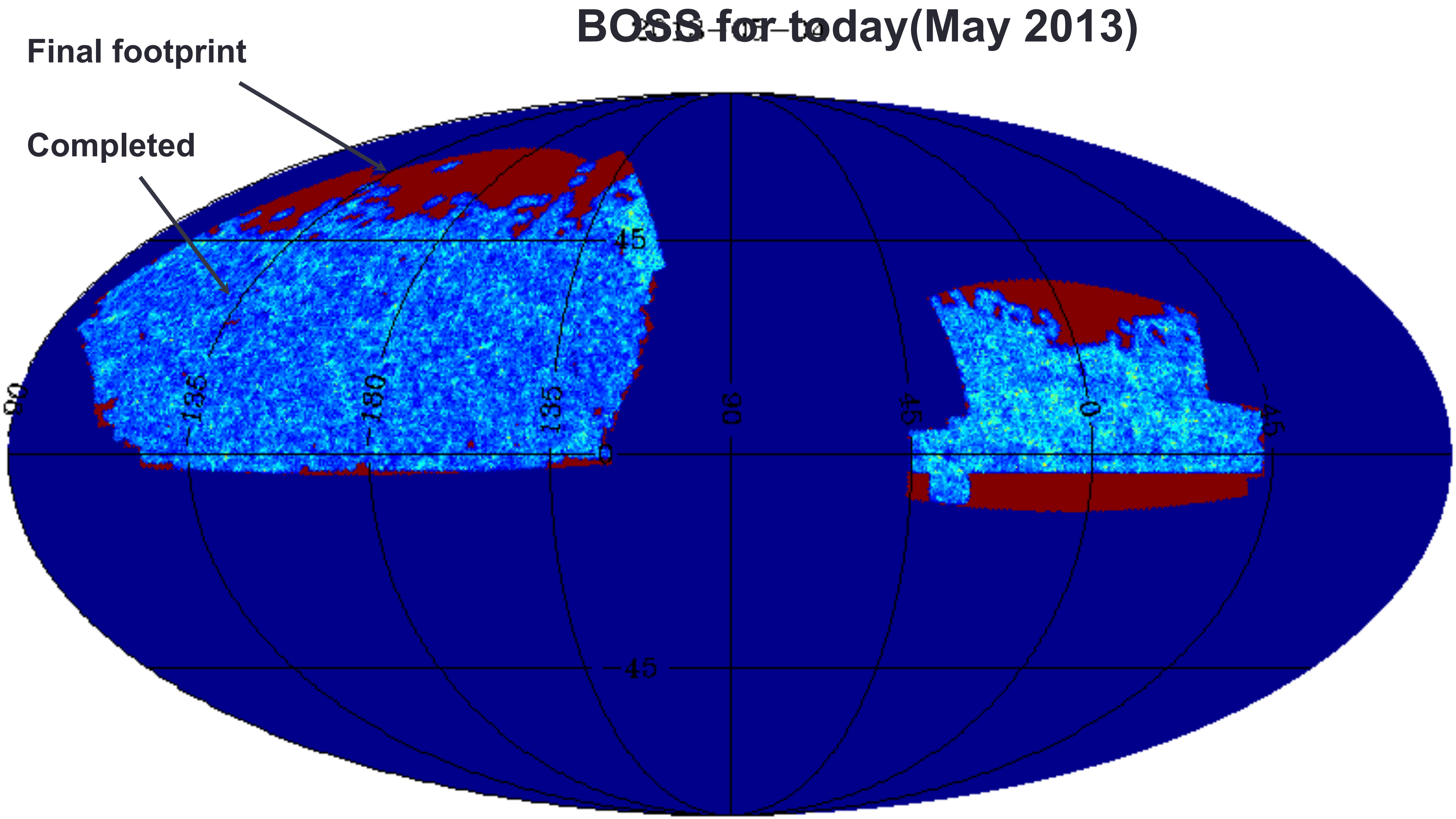
BOSS July 2012 (Data Release 10)

Final footprint

Completed

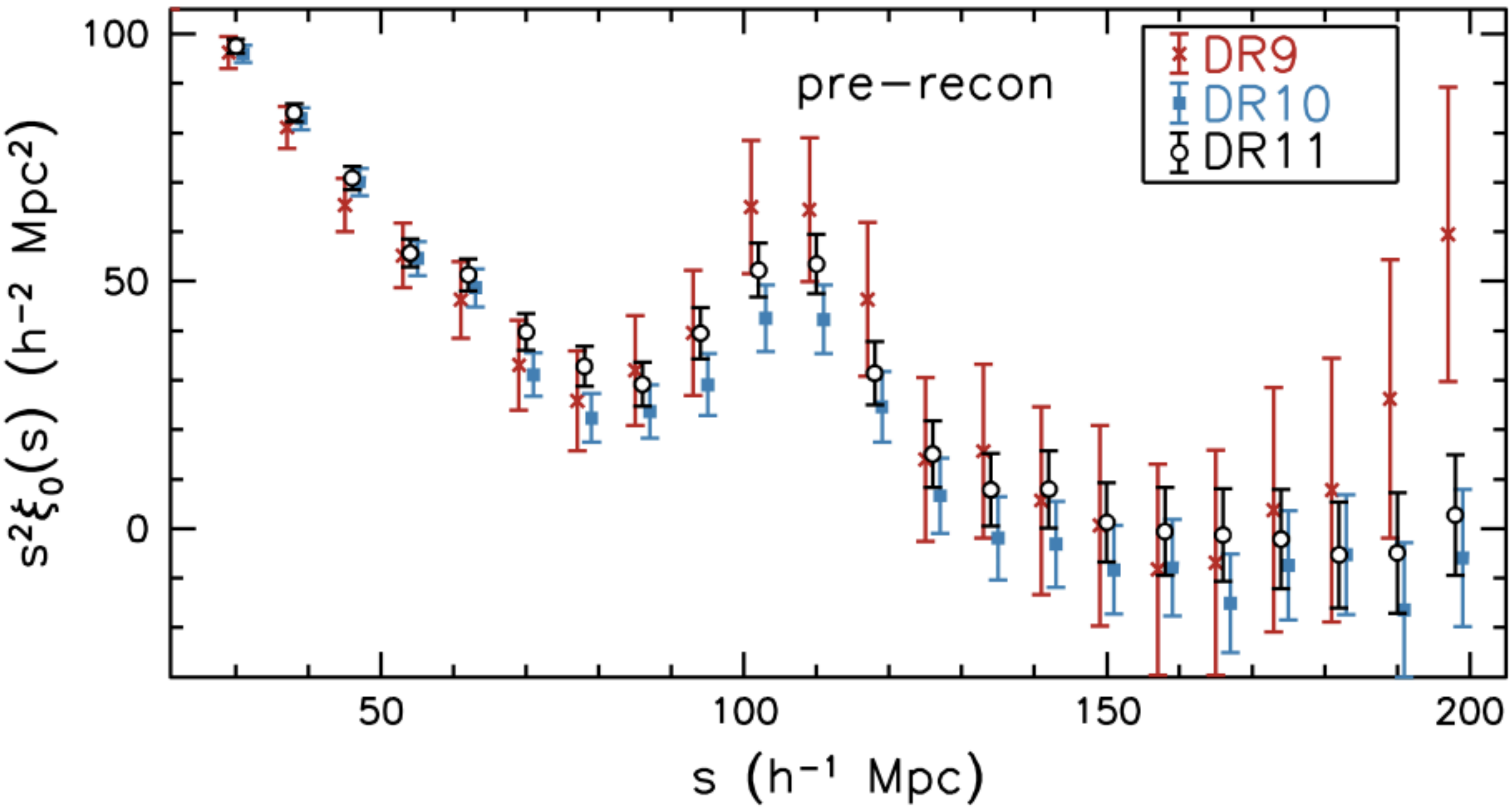


BOSS survey progress



Galaxy redshift success rate 97% (requirement was >=94%)

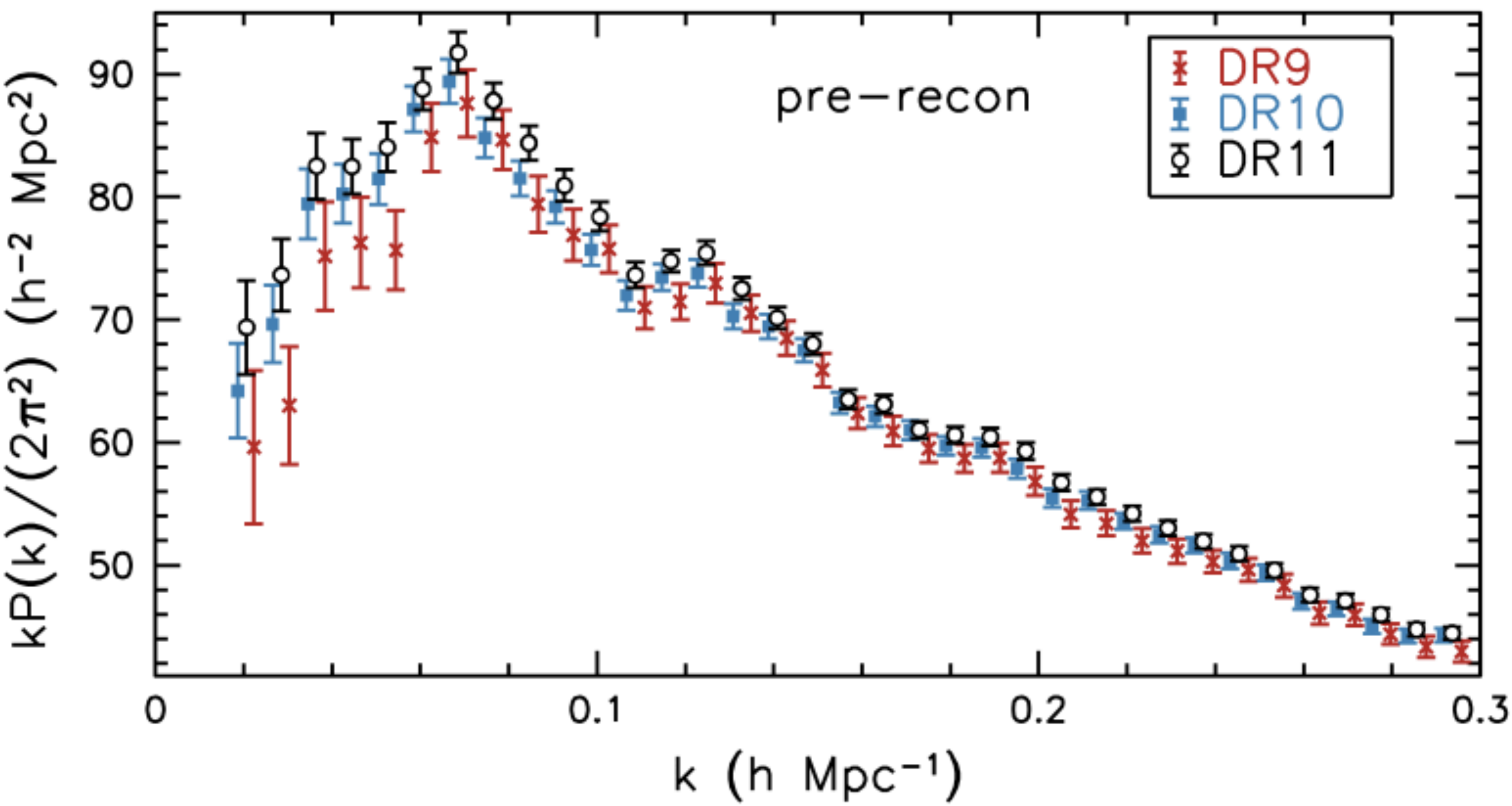
The BAO Feature clearly detected



> 7 sigma, 1% distance measurement

Anderson et al, 2014

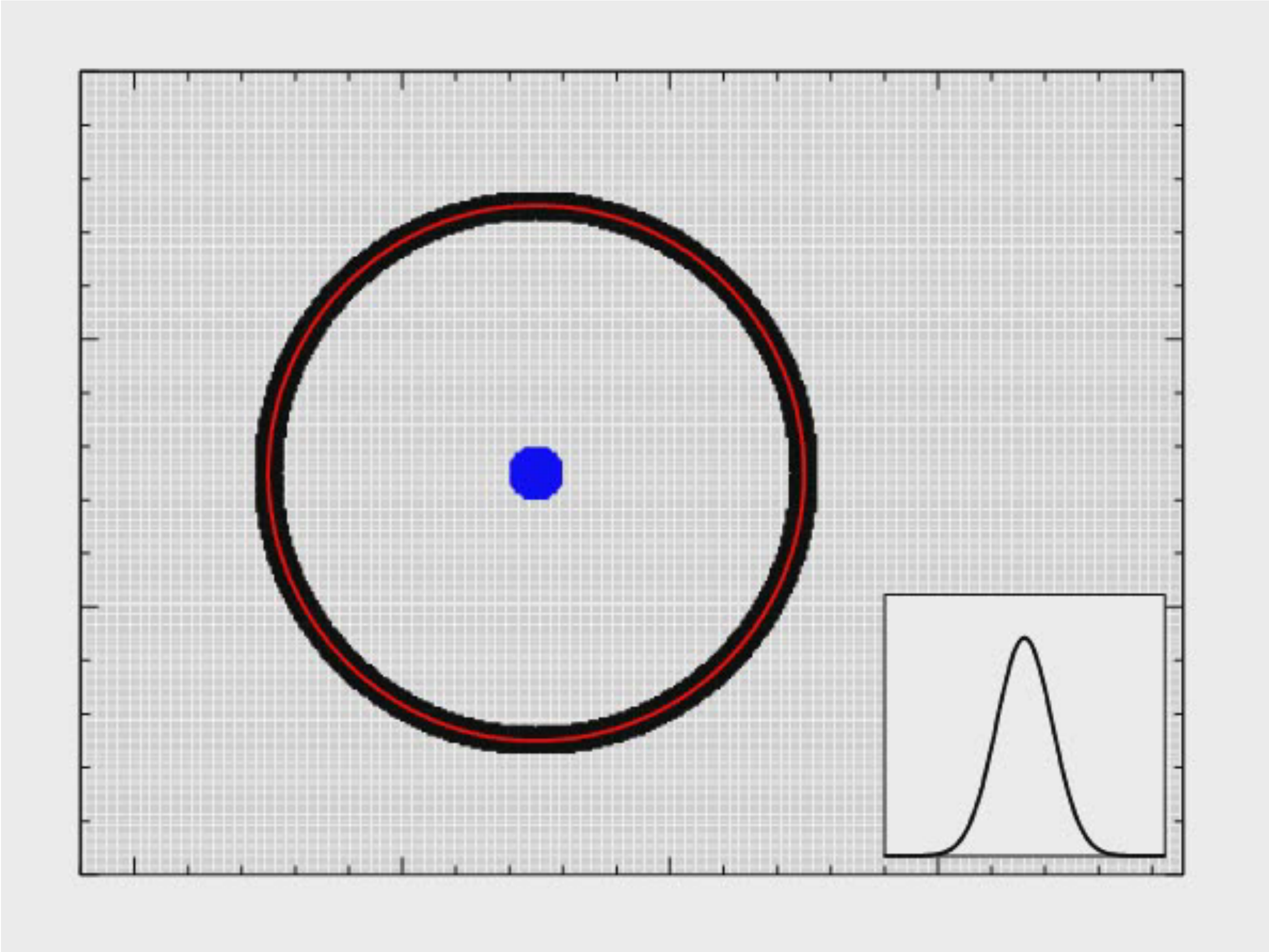
The BAO Feature clearly detected



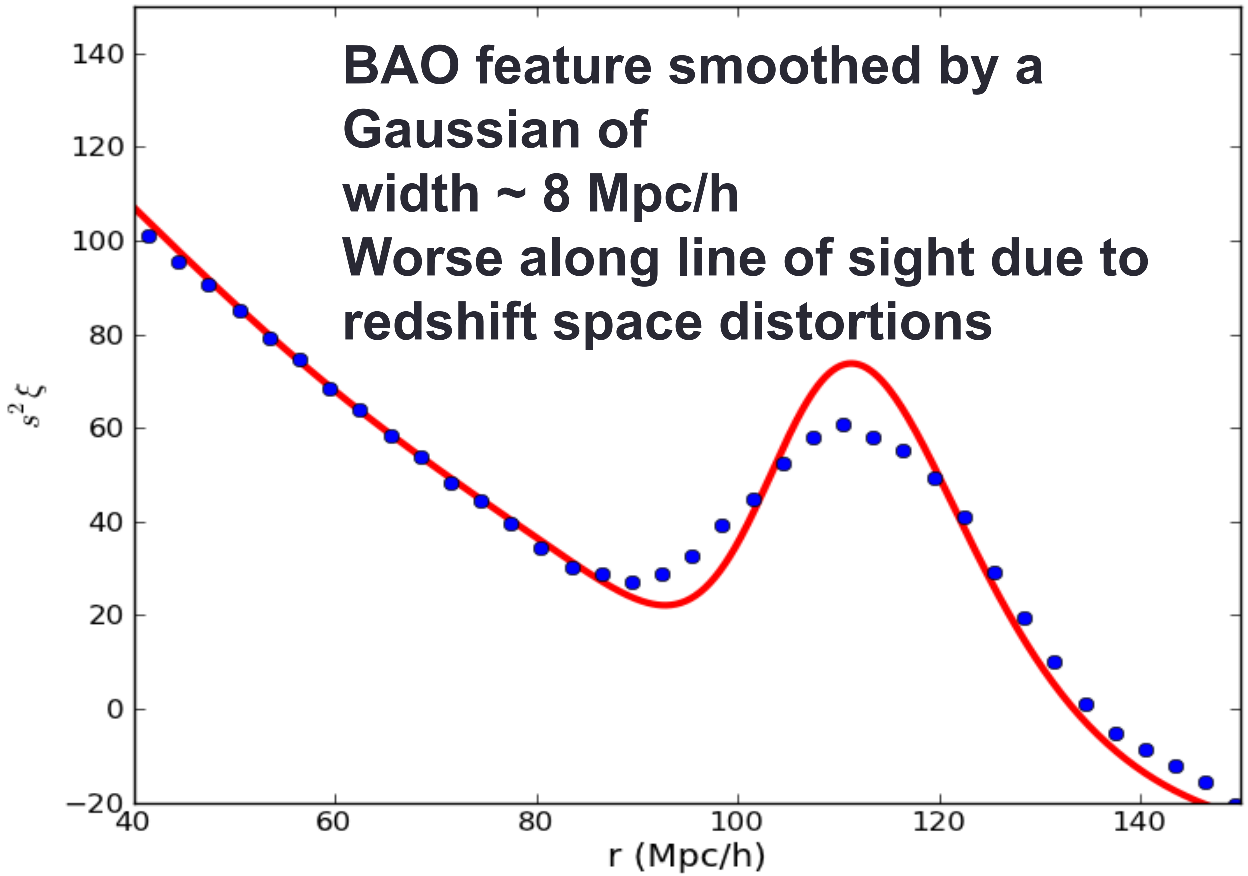
> 7 sigma, 1% distance measurement

Anderson et al, 2014

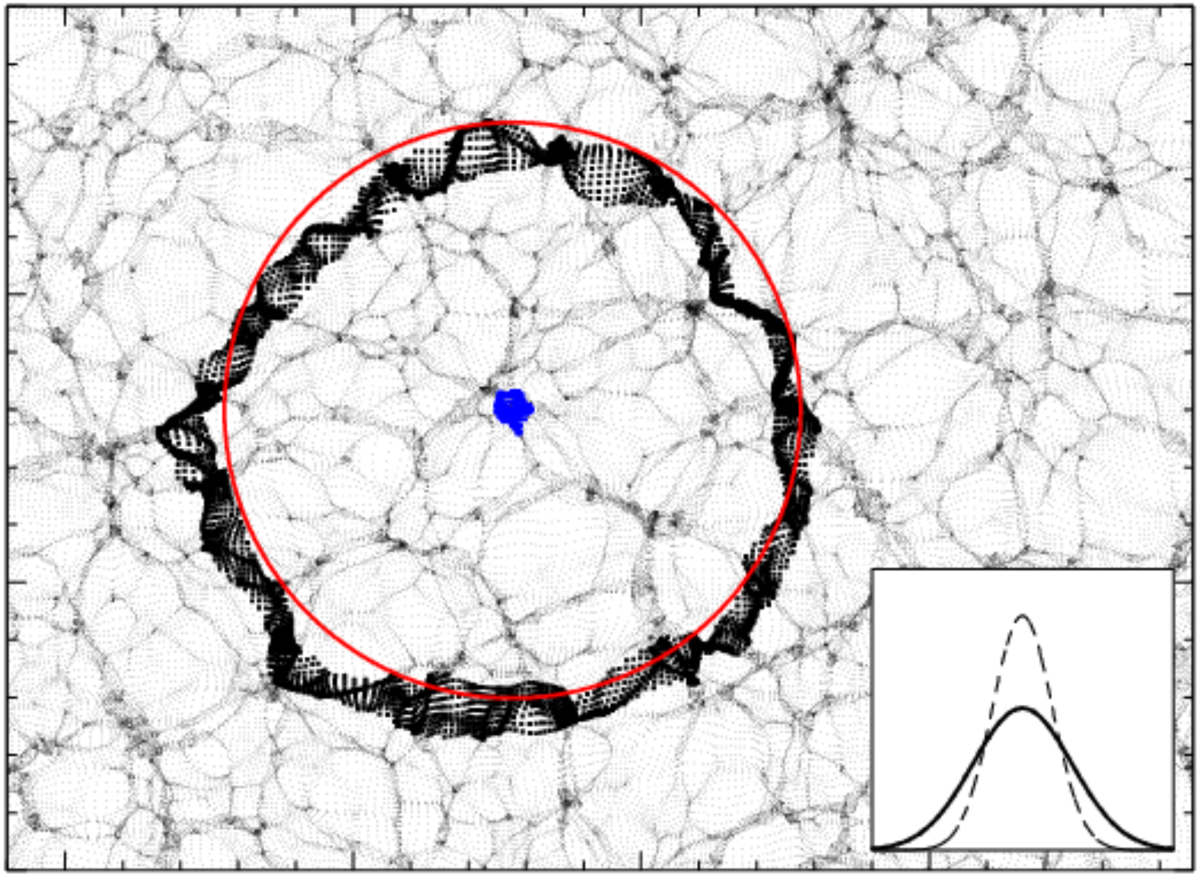
BAO and structure formation



Nonlinear evolution



Reconstruction



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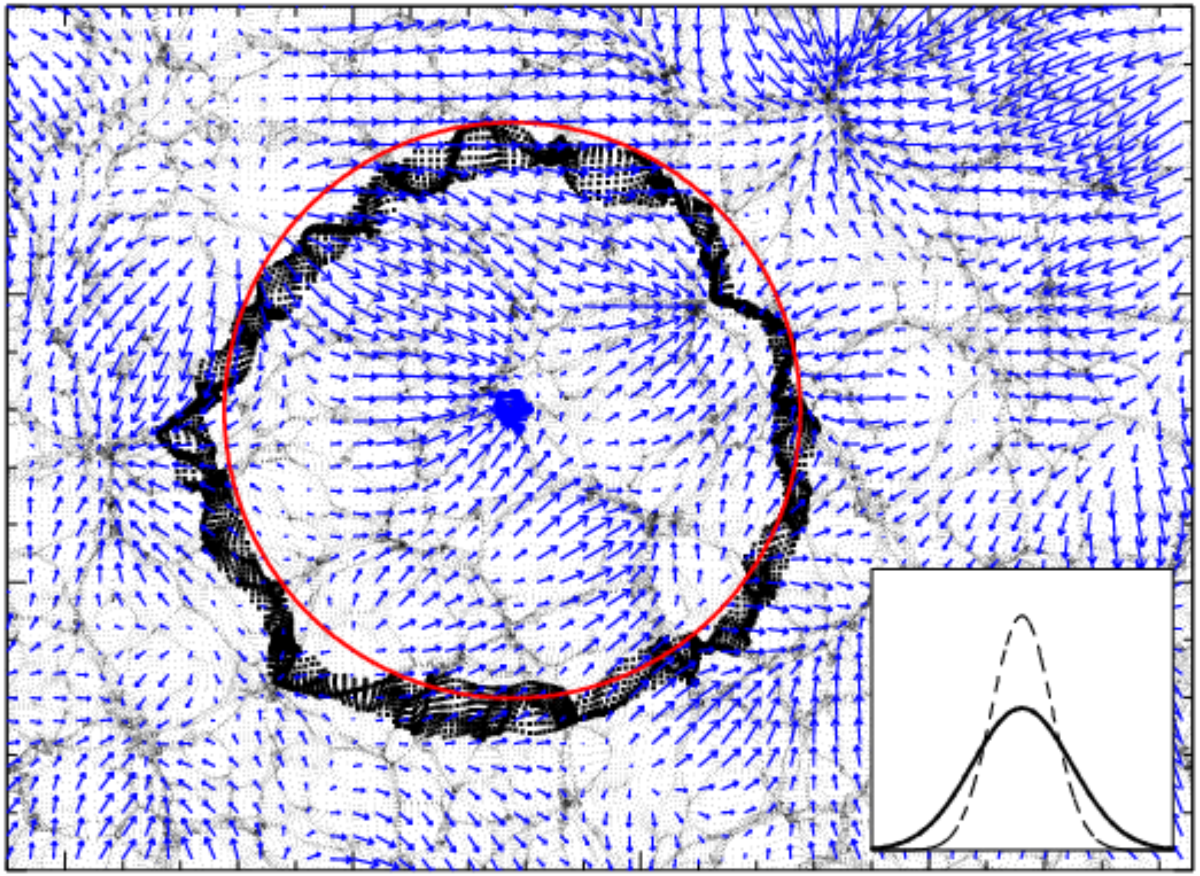

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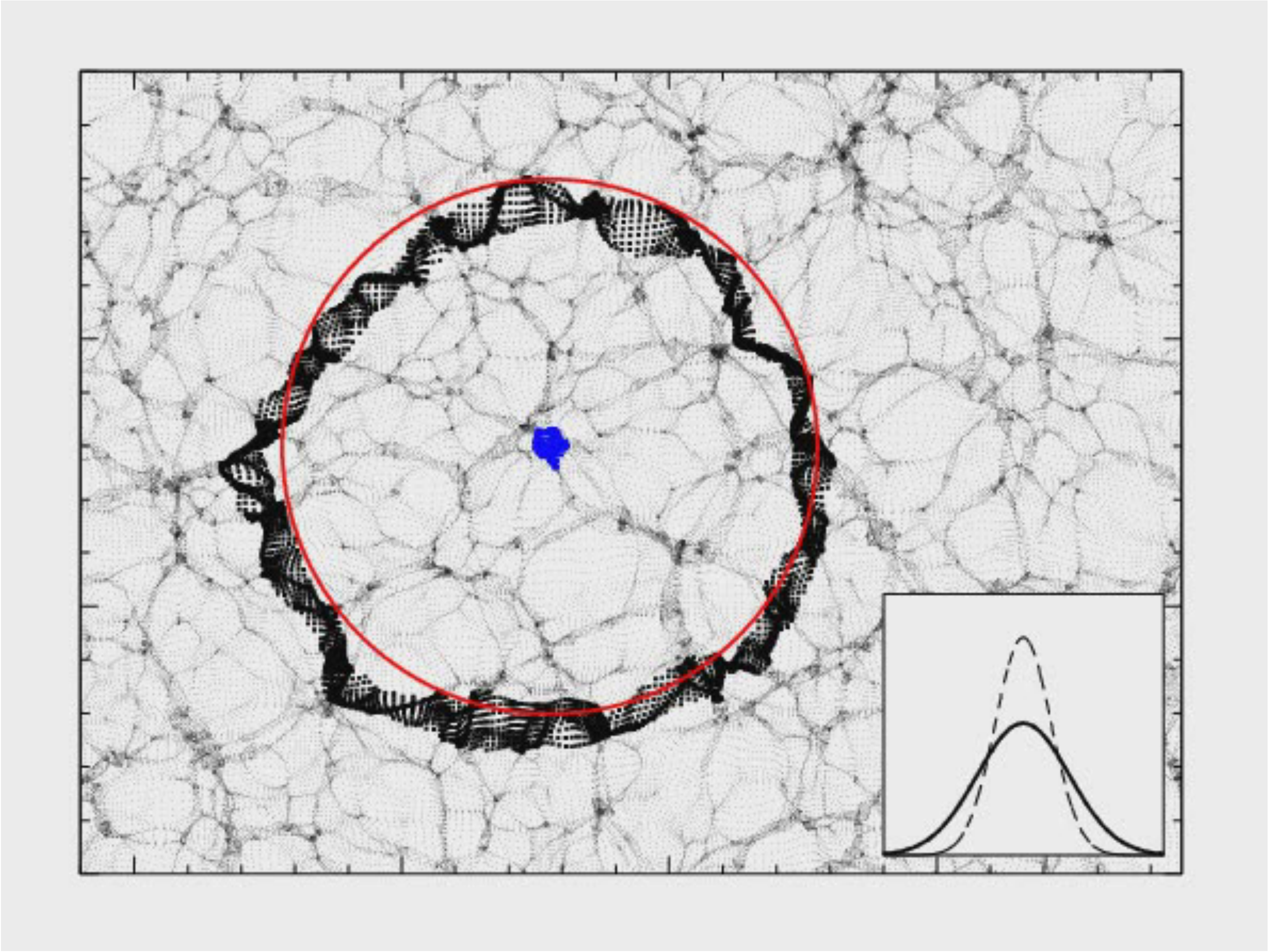
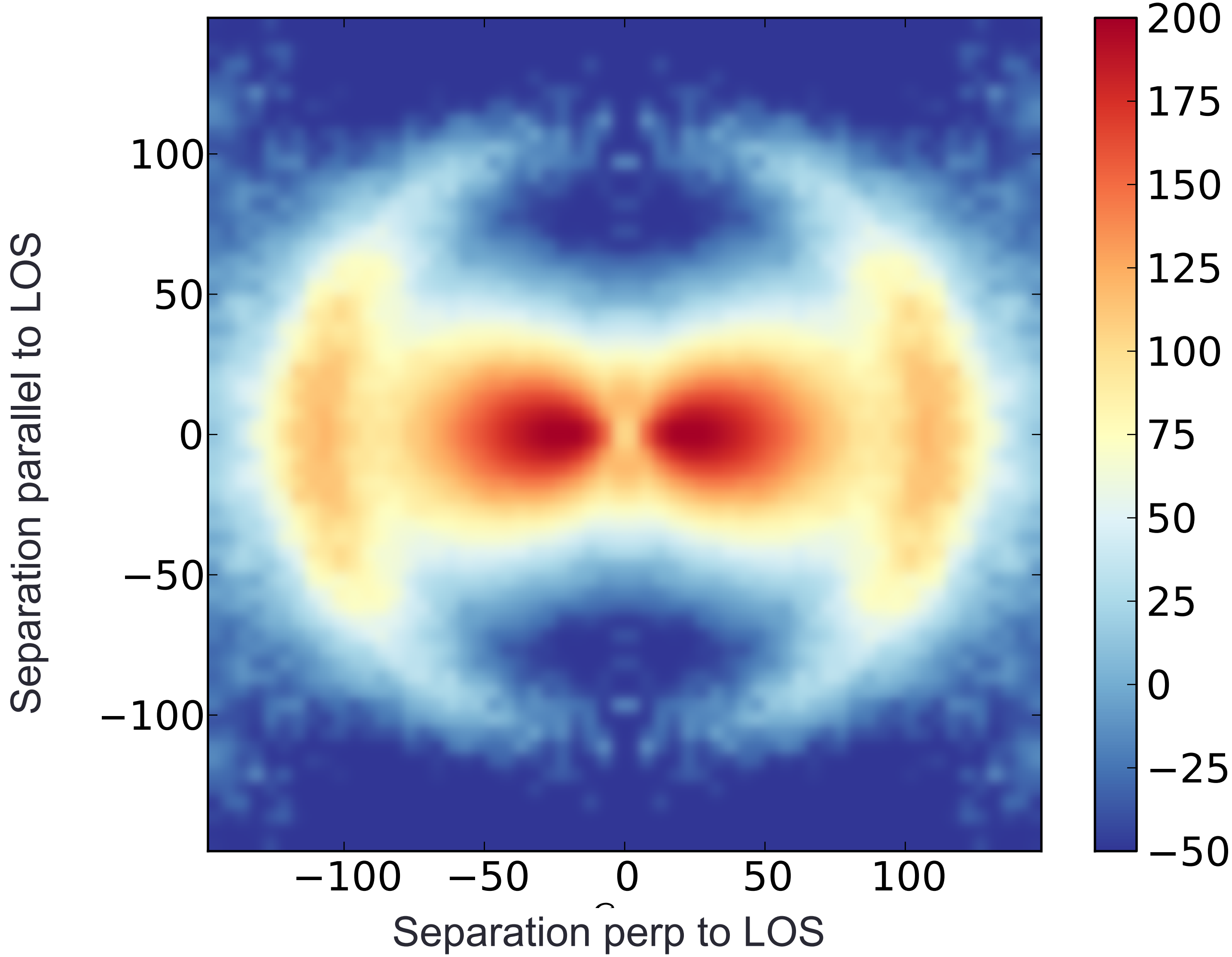


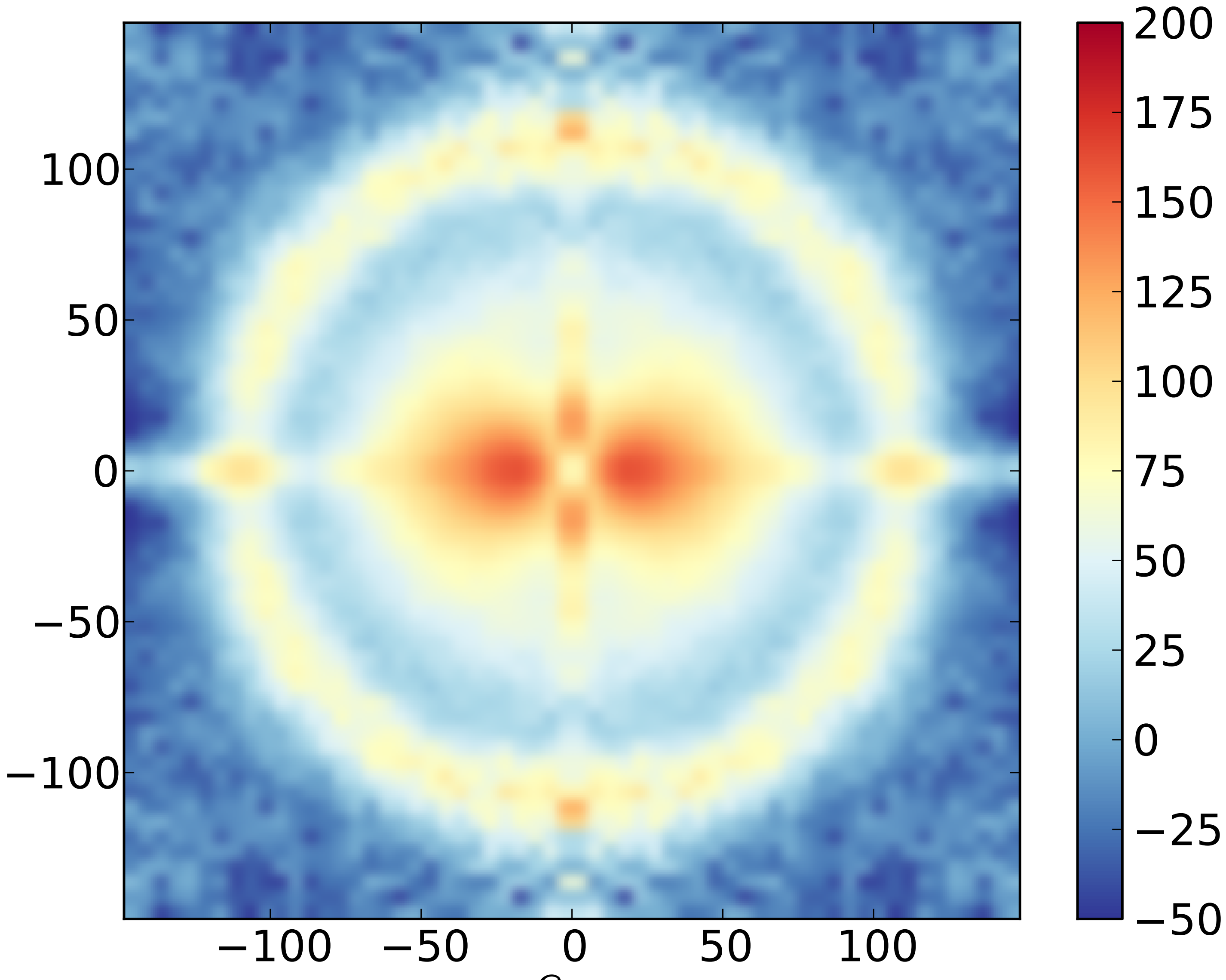
Simulations : Before

NP et al, 2012

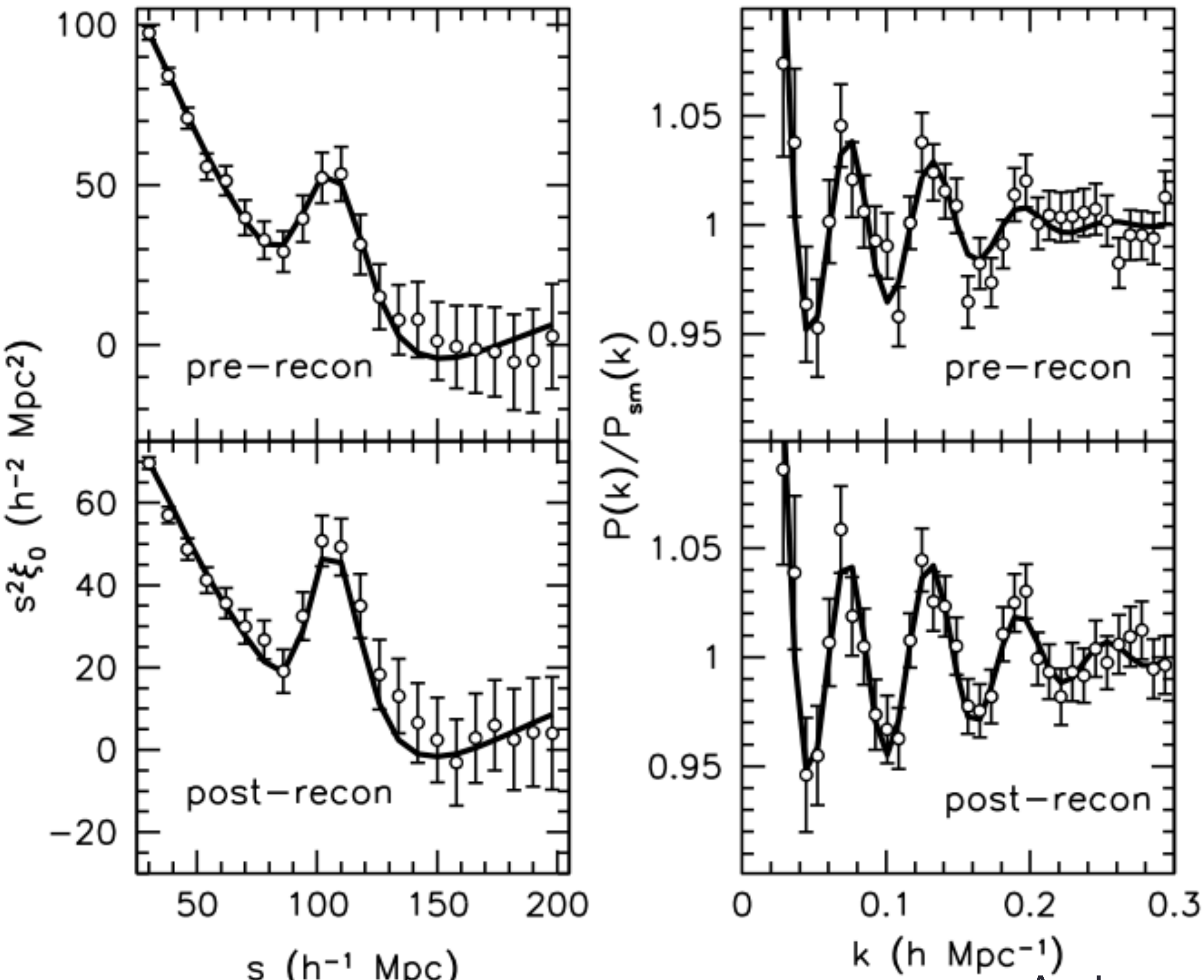


Simulations : After

NP et al, 2012

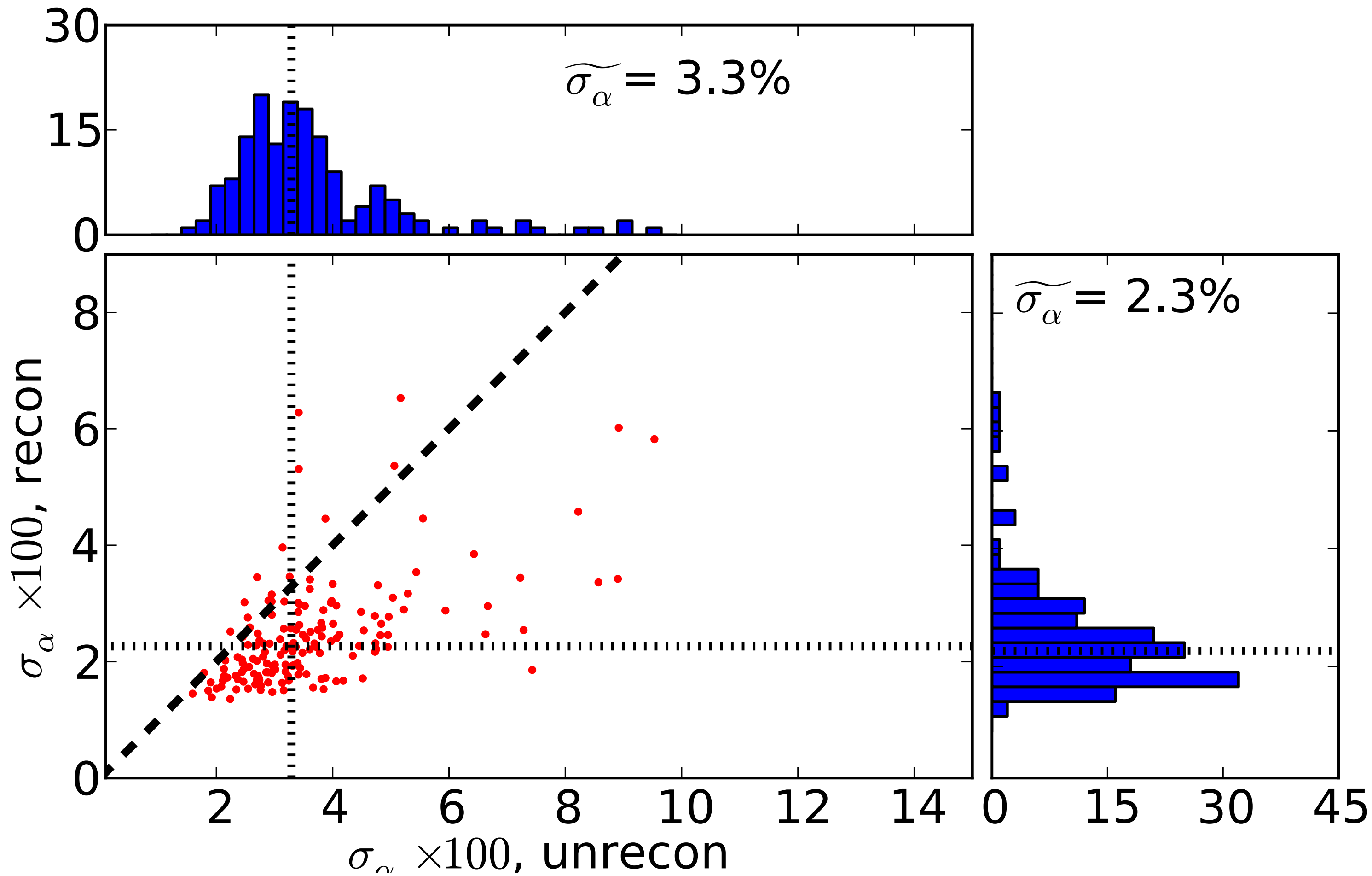


A Sharper Feature, More Oscillations

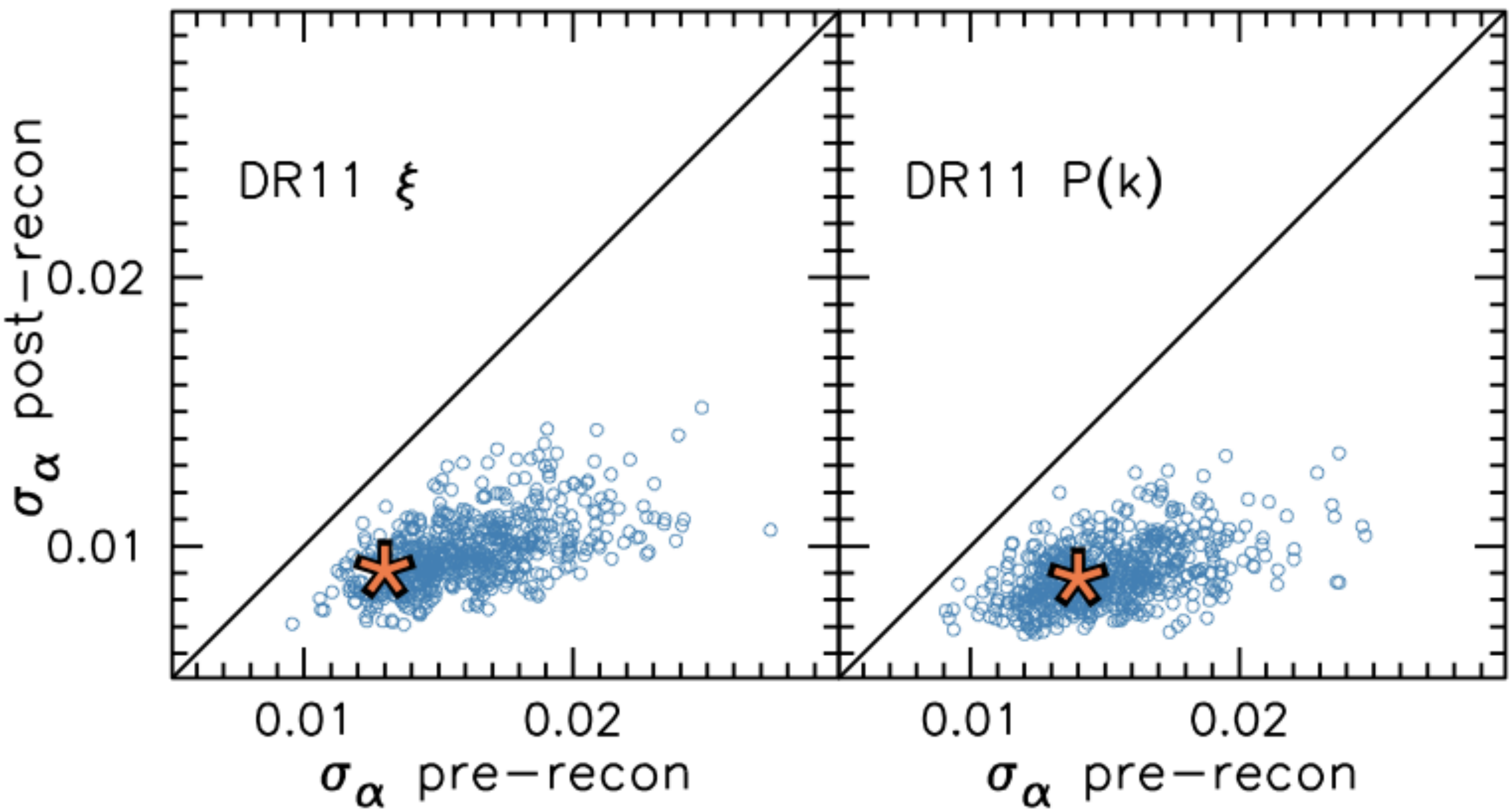


Anderson et al, 2014

Errors in SDSS-II

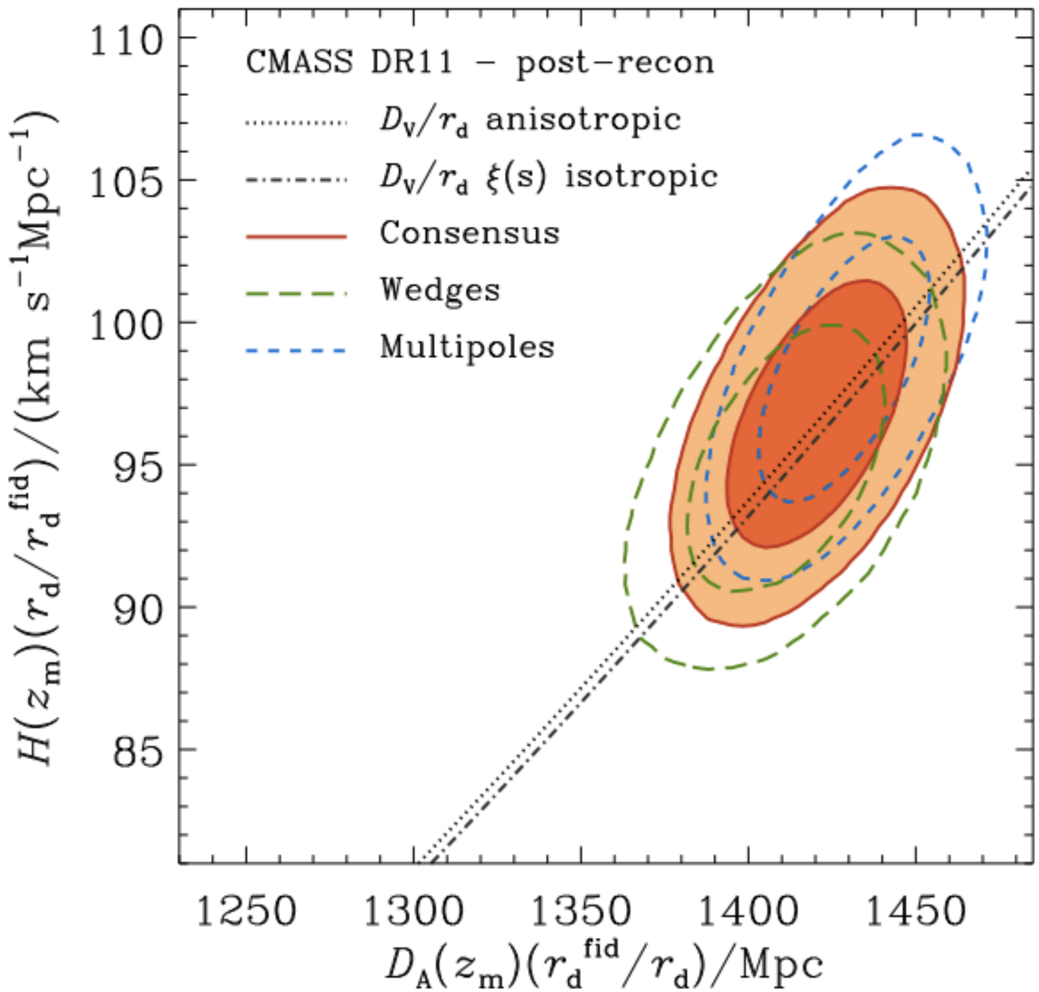


… and in DR11



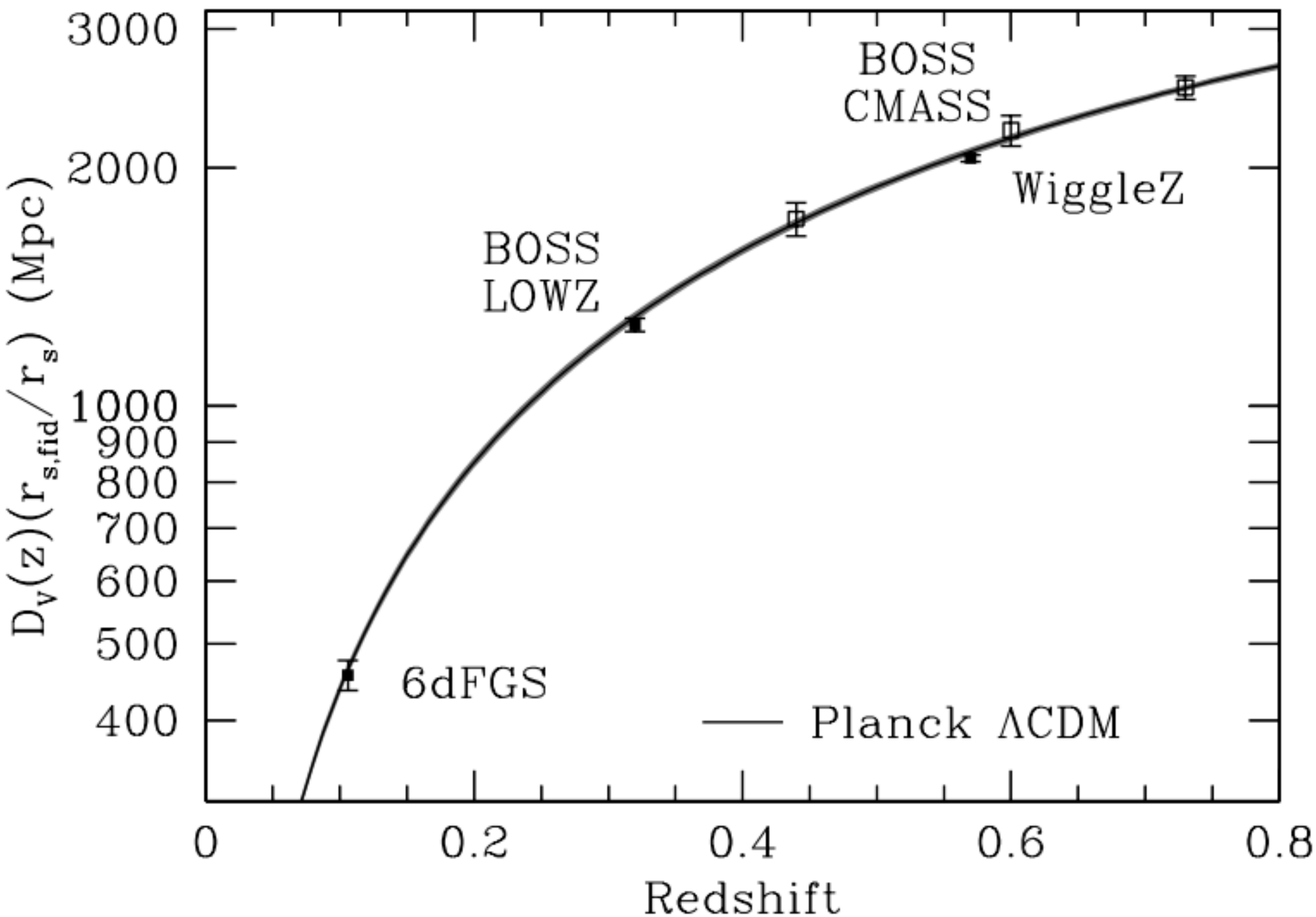
Anderson et al, 2014

BOSS measures DA and H



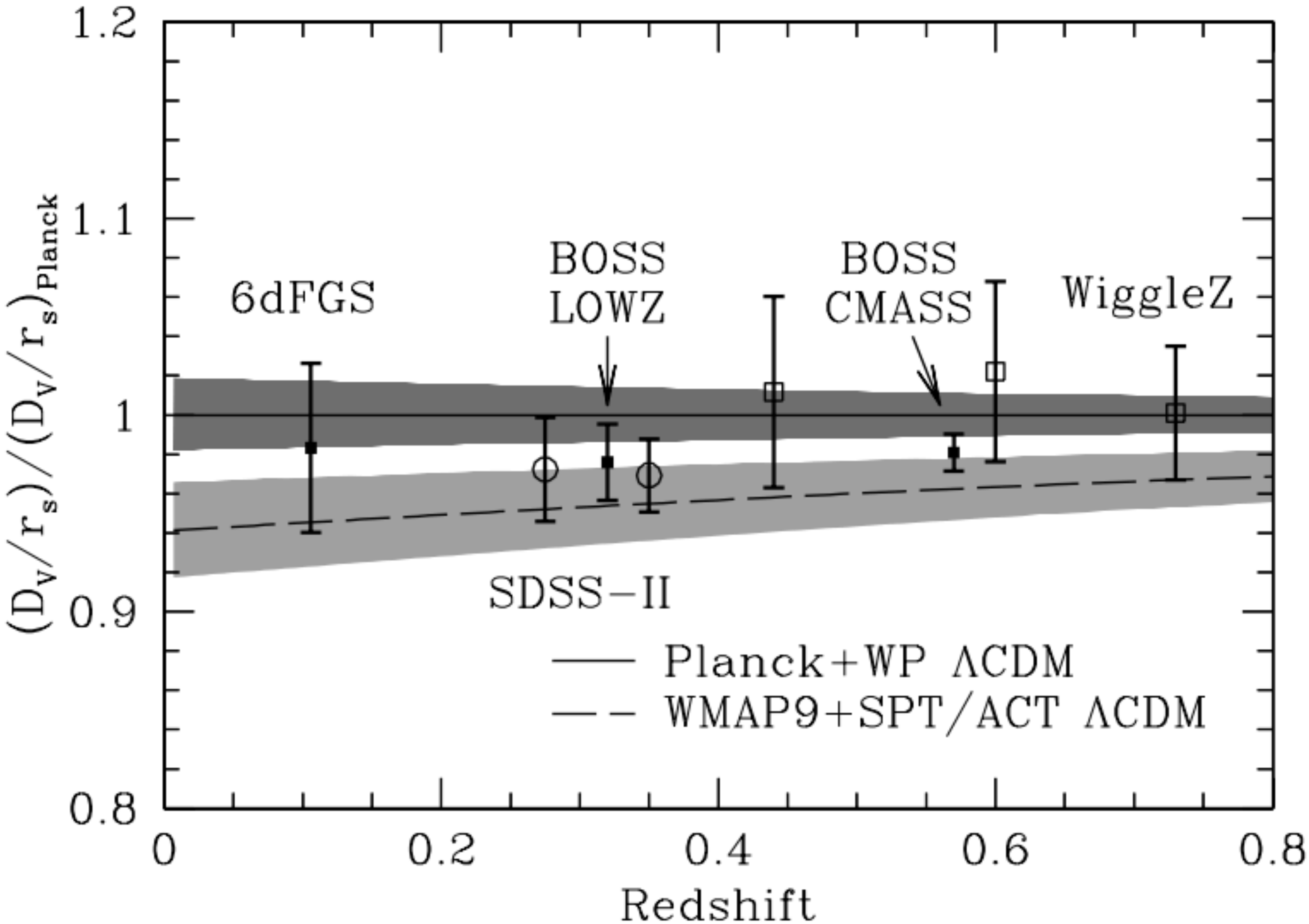
Anderson et al, 2014

BAO measure the expansion history



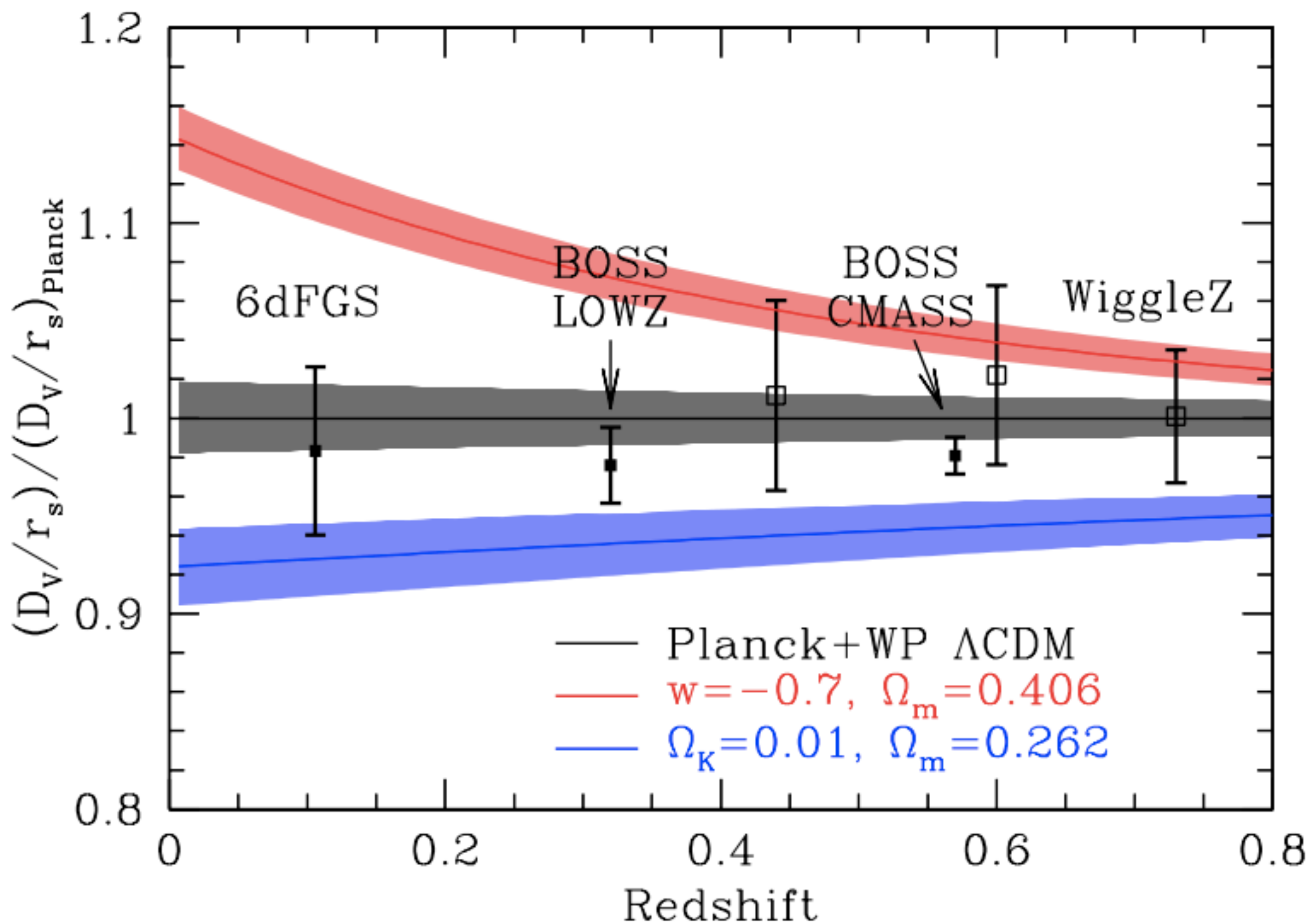
Anderson et al, 2014

A BAO Hubble diagram



Anderson et al, 2014

BAO constrains cosmology



Anderson et al, 2014

Deconstructing the Friedmann Eqn.

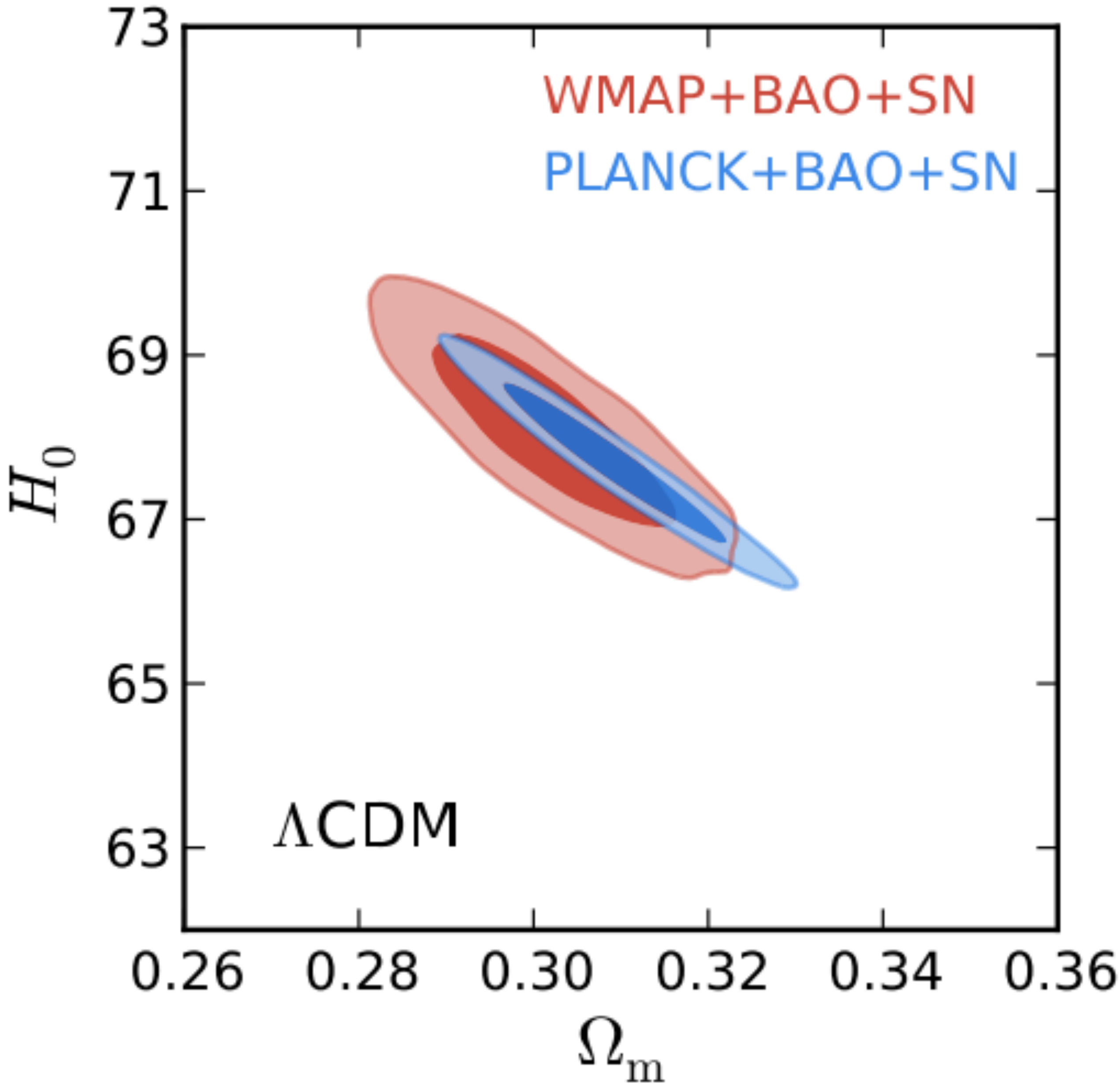
$$H^2(a) = H_0^2 \left[\Omega_R a^{-4} + \Omega_M a^{-3} + \right.$$

$$\left. \Omega_k a^{-2} + \Omega_{DE} \exp\left\{ 3 \int_a^1 \frac{da'}{a'} [1 + w(a')] \right\} \right]$$

The matter density

$$H^2(a) = H_0^2 \Big[\Omega_R a^{-4} + \Omega_M a^{-3} +$$

$$\Omega_k a^{-2} + \Omega_{DE} \exp \left\{ 3 \int_a^1 \frac{da'}{a'} \left[1 + w(a') \right] \right\} \Big]$$

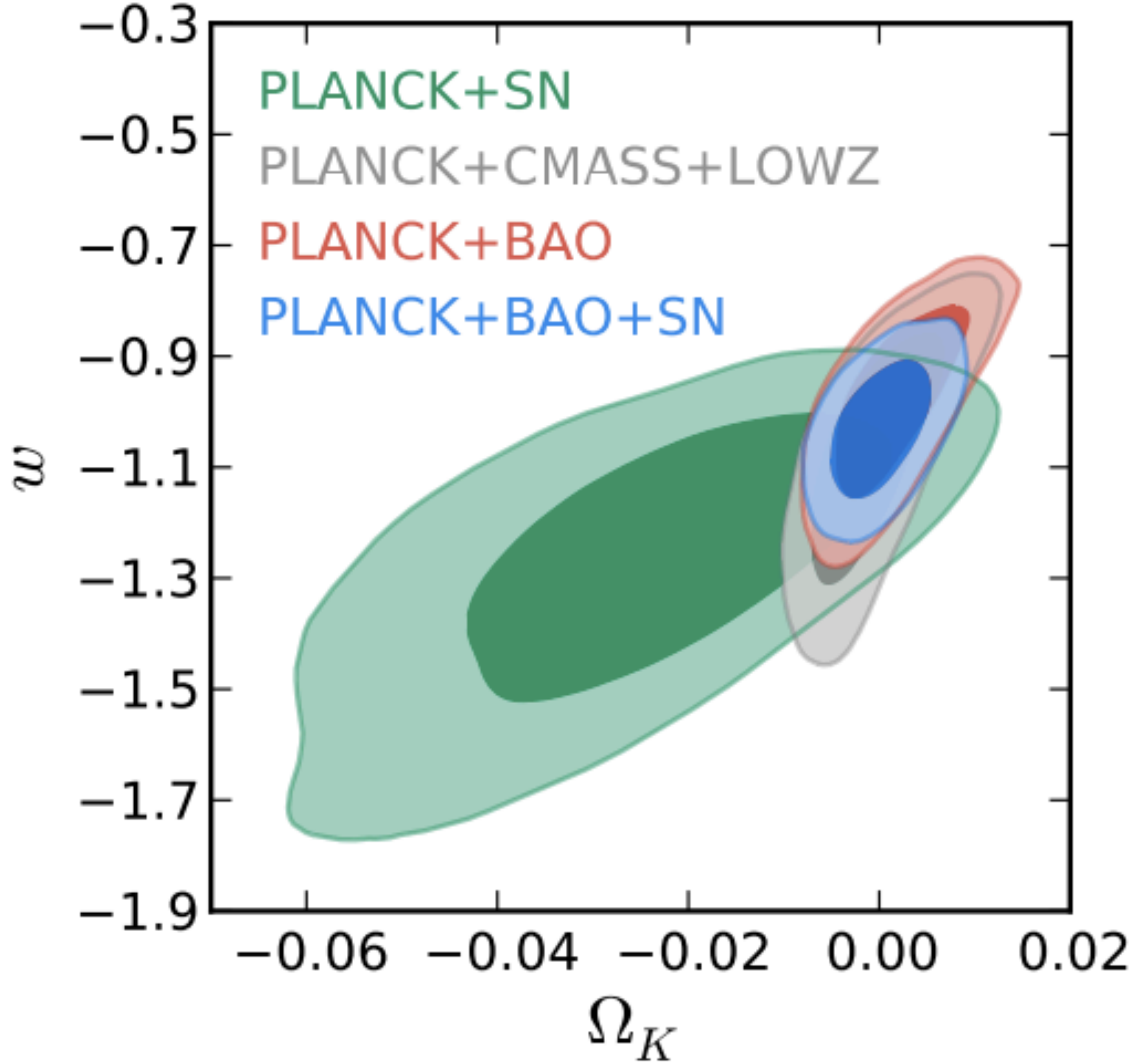


Anderson et al, 2014

Measuring the curvature

$$H^2(a) = H_0^2\left[\Omega_R a^{-4} + \Omega_M a^{-3} + \right.$$

$$\left. \Omega_k a^{-2} + \Omega_{DE} \exp\left\{3\int_a^1 \frac{da'}{a'}\left[1 + w(a')\right]\right\}\right]$$

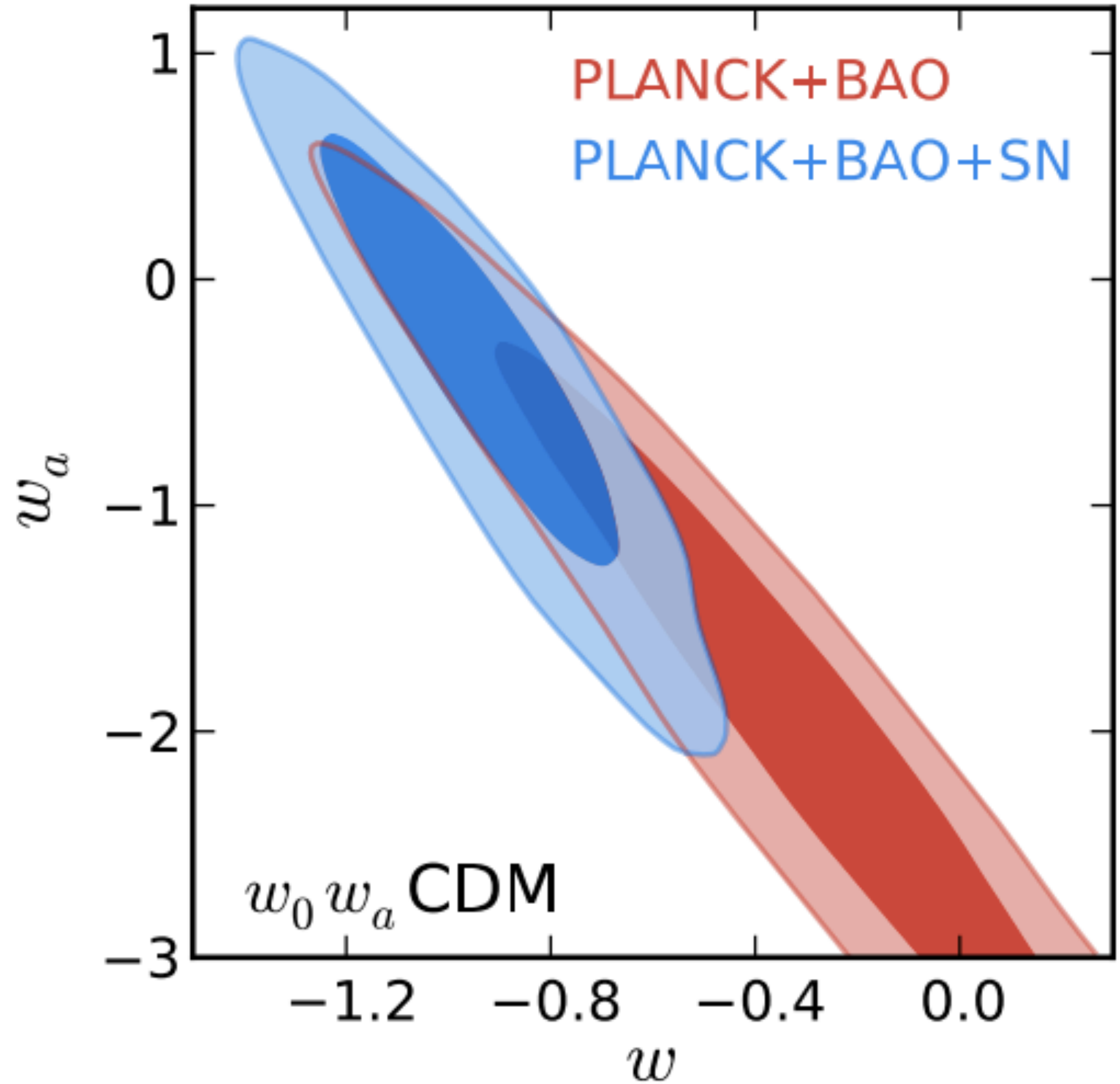


Anderson et al, 2014

Measuring the eqn of state

$$H^2(a) = H_0^2 \Big[\Omega_R a^{-4} + \Omega_M a^{-3} +$$

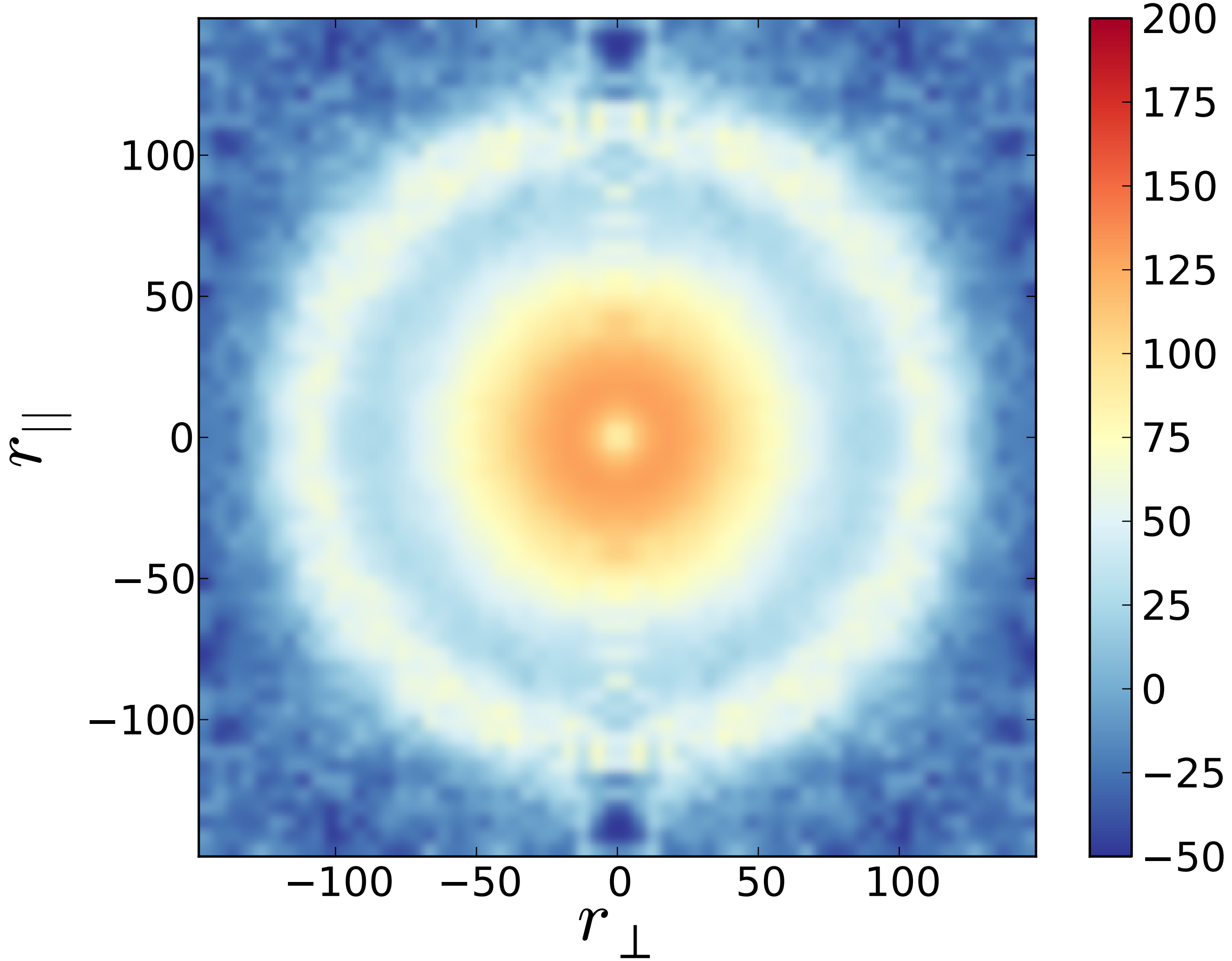
$$\Omega_k a^{-2} + \Omega_{DE} \exp \left\{ 3 \int_a^1 \frac{da'}{a'} \left[1 + w(a') \right] \right\} \Big]$$



Anderson et al, 2014

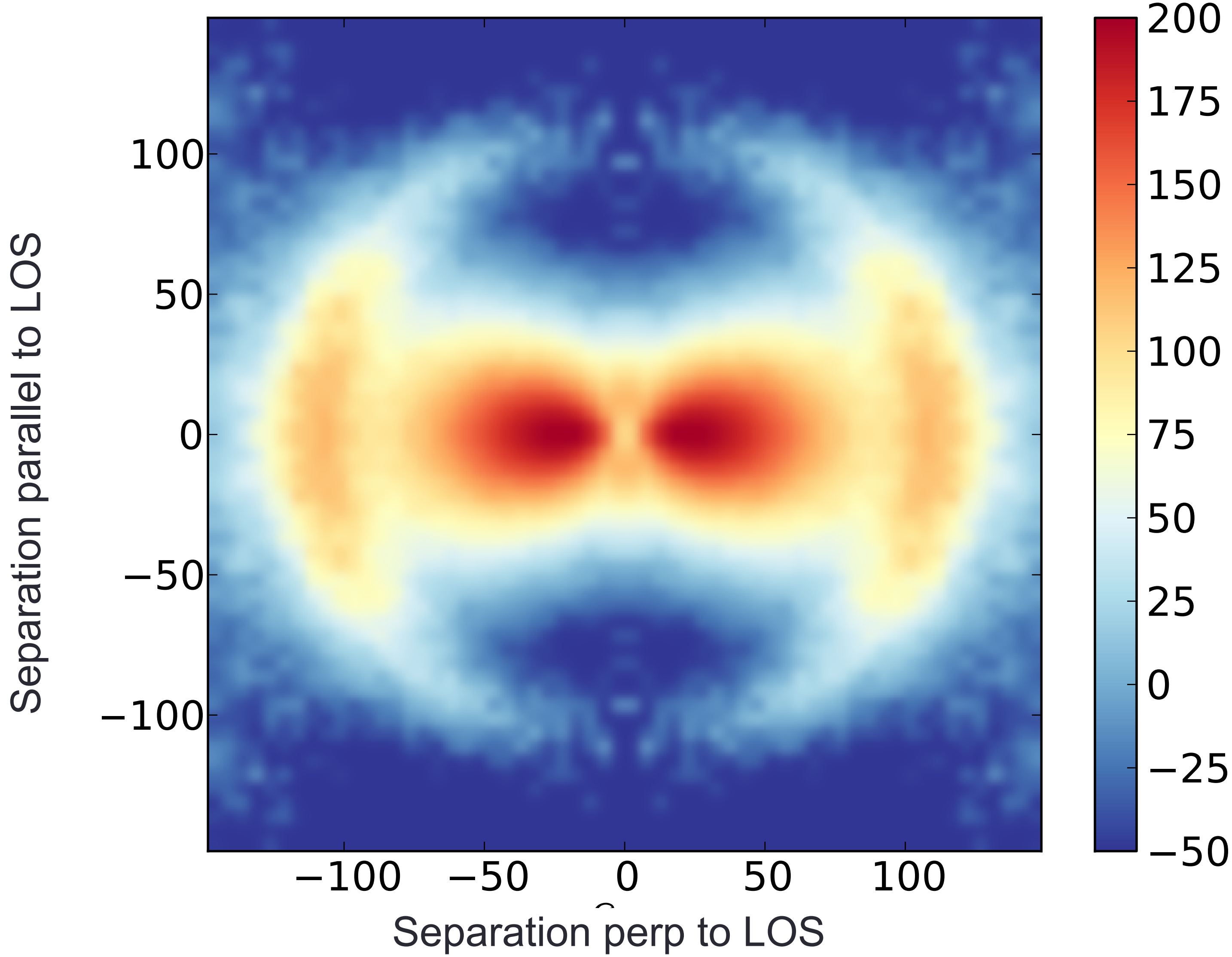
Measuring BAO : A Guide

Simulations : Real Space



Simulations : Before

NP et al, 2012



Measuring BAO : I

- The data consist of angles and redshifts. To convert to comoving coordinates, we need to assume a fiducial cosmology.
- Is the BAO feature at the correct place in this cosmology?
  - That determines the distance scale
- We can do this perpendicular and parallel to the LOS.
  - Measure DA and H
- An alternative parametrization : dilations and warping

Definitions

Define an angle averaged distance.

$$D_V(z) = \left[(1+z)^2 D_A^2(z) \frac{cz}{H(z)} \right]^{1/3}$$

Measure shifts in the BAO scale.
Note the scaling with the sound horizon

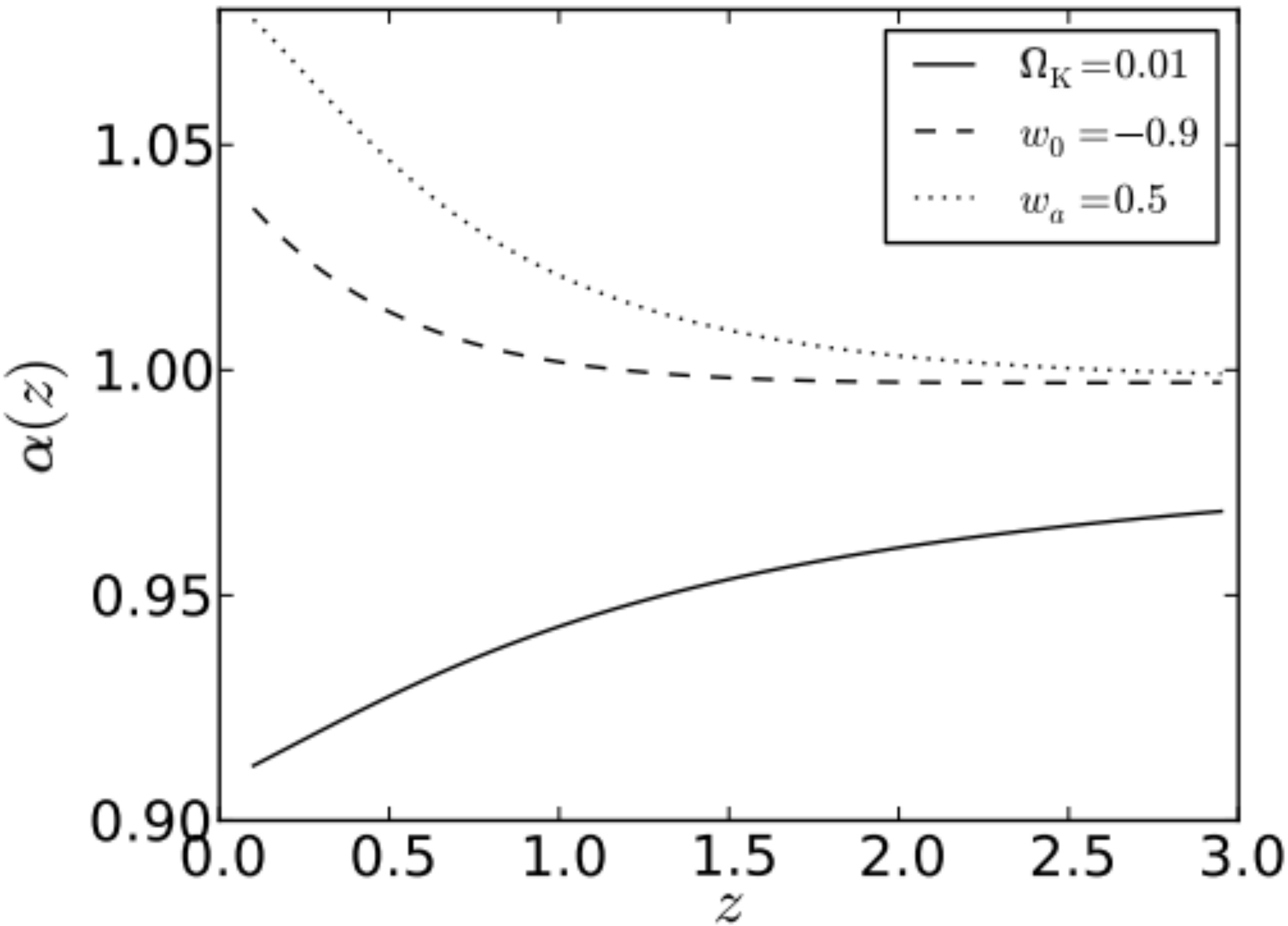
$$\alpha = \frac{D_V(z)/r_s}{D_{V,f}(z)/r_{s,f}}$$

The Alcock-Paczynski (AP) effect

$$1 + \epsilon = \left[\frac{H_f(z)}{H(z)} \frac{D_{A,f}(z)}{D_A(z)} \right]^{1/3} .$$

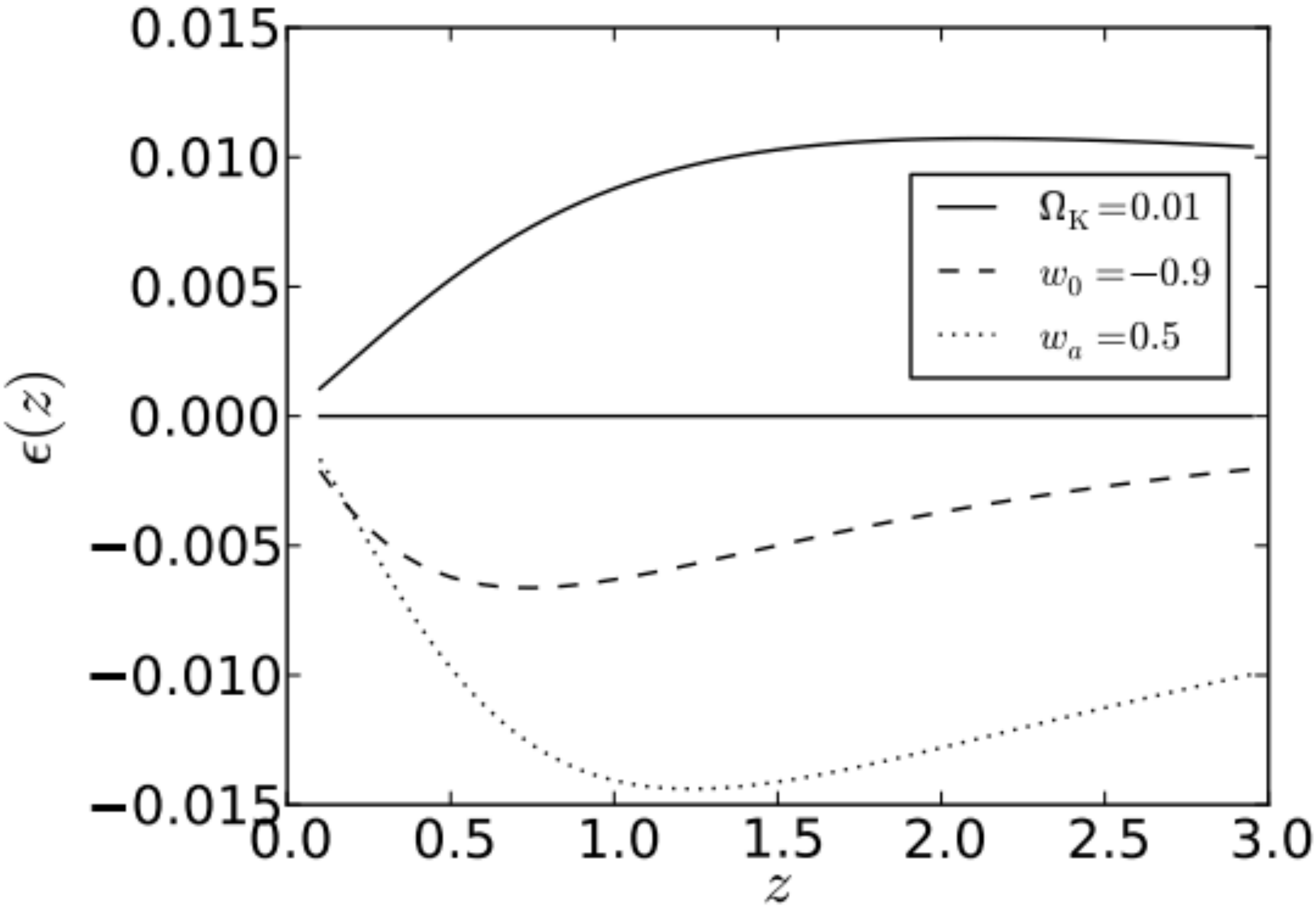
Corrections to the cosmology involve alpha (dilations) and warping (epsilon).
Alpha=1, epsilon=0 => true cosmology
How do these effect the BAO feature?

Relative importance of dilations and warping



Xu et al, 2012

Relative importance of dilations and warping



Xu et al, 2012

The Anisotropic BAO signal

$$
\begin{aligned}
r^2 &= r_\parallel^2 + r_\perp^2 & r'_\parallel &= \alpha(1+\epsilon)^2 r_\parallel \\
\mu^2 &= \cos^2\theta = \frac{r_\parallel^2}{r^2} & r'_\perp &= \alpha(1+\epsilon)^{-1} r_\perp .
\end{aligned}
$$

$$
\begin{aligned}
r' &= \alpha\sqrt{(1+\epsilon)^4 r_\parallel^2 + (1+\epsilon)^{-2} r_\perp^2} \\
&= \alpha r[1 + 2\epsilon L_2(\mu)] + \mathcal{O}(\epsilon^2)
\end{aligned}
$$

$$
\begin{aligned}
\mu'^2 &= \frac{\alpha^2(1+\epsilon)^4 r_\parallel^2}{\alpha^2(1+\epsilon)^4 r_\parallel^2 + \alpha^2(1+\epsilon)^{-2} r_\perp^2} \\
&= \mu^2 + 6\epsilon(\mu^2 - \mu^4) + \mathcal{O}(\epsilon^2).
\end{aligned}
$$

Primed coordinates represent true cosmology, unprimed are fiducial

Xu et al, 2012

Multipole expansions

$$\xi(\vec{r'}) = \sum_{\ell'=0}^{\infty} \xi_{\ell'}(r')L_{\ell'}(\mu')$$

$$P_{\ell,t}(k) = \frac{2\ell+1}{2}\int_{-1}^{1} P_t(k,\mu)L_\ell(\mu)d\mu,$$

$$\xi_{\ell,t}(r) = i^\ell \int \frac{k^3 d\log(k)}{2\pi^2} P_{\ell,t}(k)j_\ell(kr).$$

In redshift space, in linear theory, only l=0,2,4 exist

Xu et al, 2012

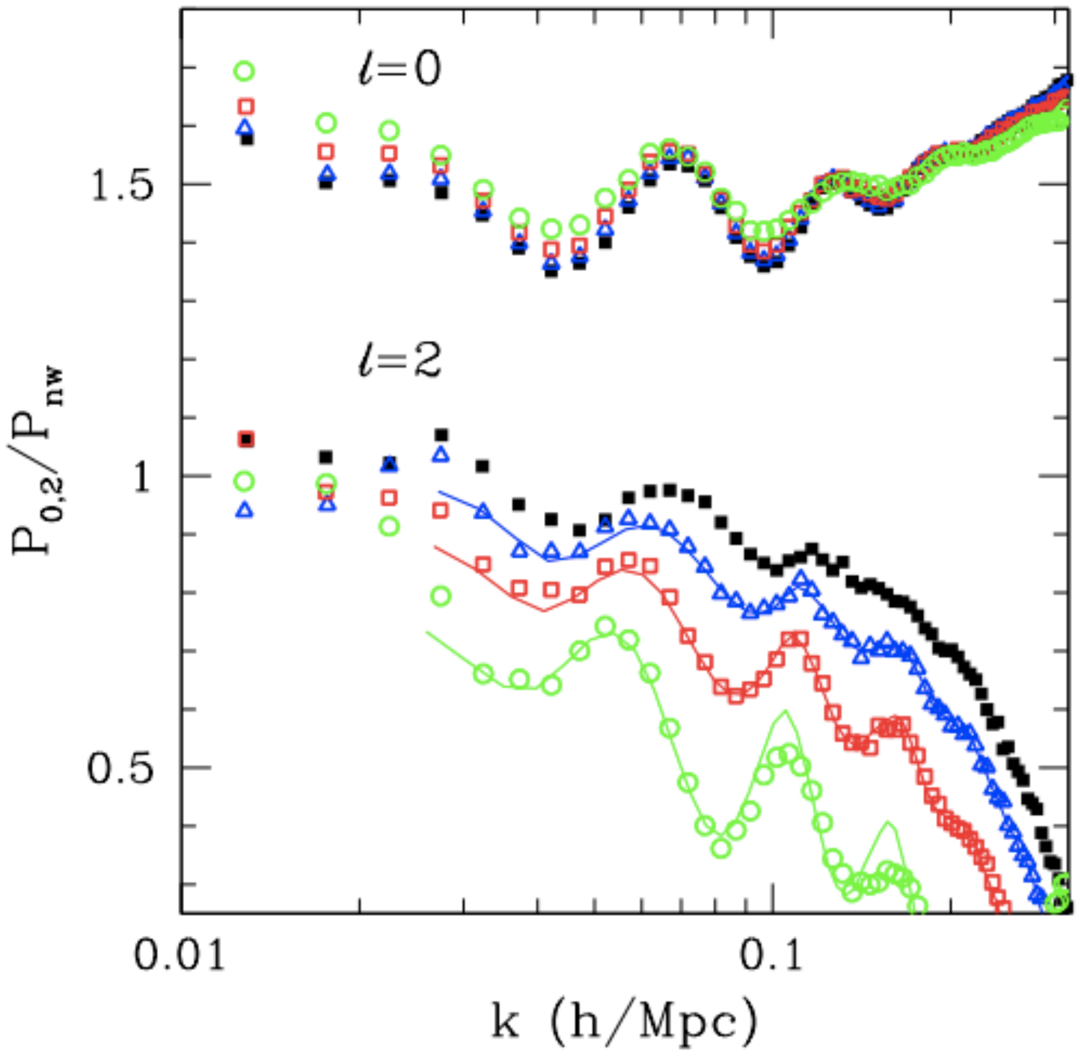
Effects on correlation function multipoles

$$
\begin{aligned}
\xi_0(r) &= \xi_0(\alpha r) + \frac{2}{5}\epsilon\left[3\xi_2(\alpha r) + \frac{d\xi_2(\alpha r)}{d\log(r)}\right] \\
\xi_2(r) &= 2\epsilon\frac{d\xi_0(\alpha r)}{d\log(r)} + \left(1+\frac{6}{7}\epsilon\right)\xi_2(\alpha r) + \frac{4}{7}\epsilon\frac{d\xi_2(\alpha r)}{d\log(r)} \\
&\quad + \frac{4}{7}\epsilon\left[5\xi_4(\alpha r) + \frac{d\xi_4(\alpha r)}{d\log(r)}\right],
\end{aligned}
$$

Mixing of monopole and quadrupole

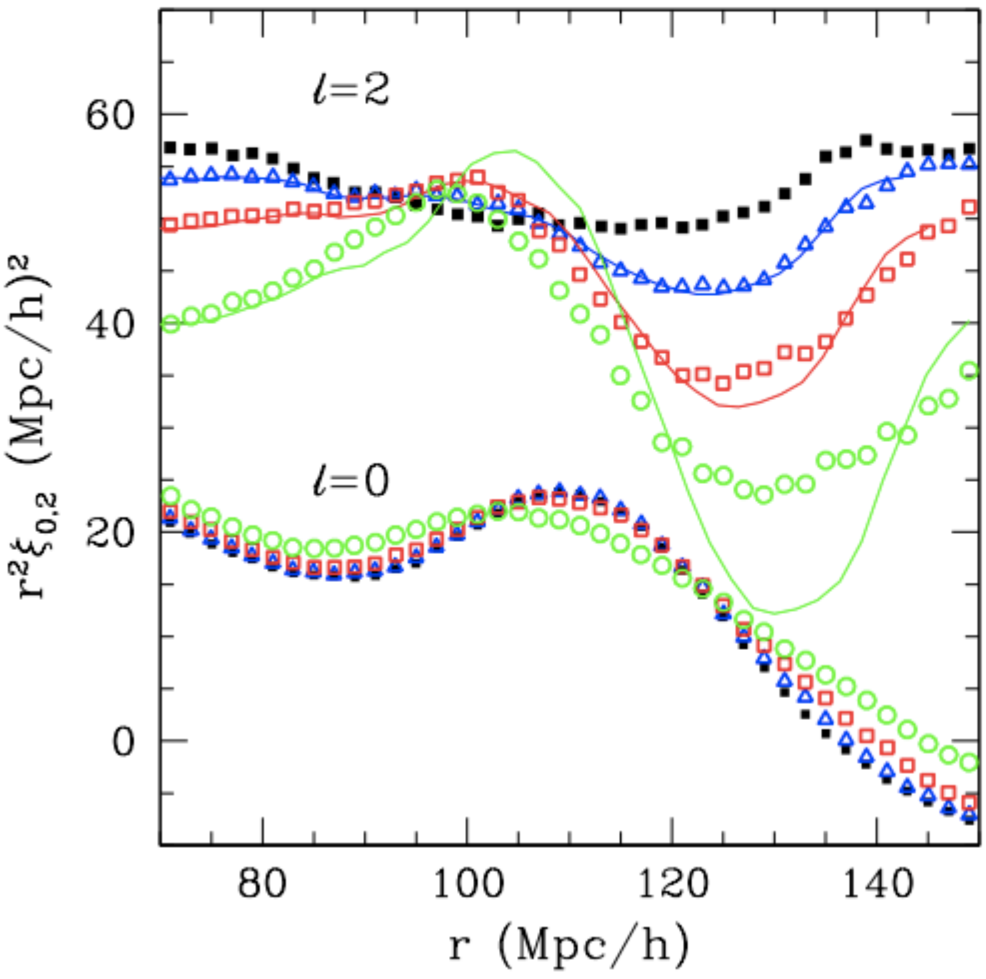
Xu et al, 2012

Effects on the power spectrum

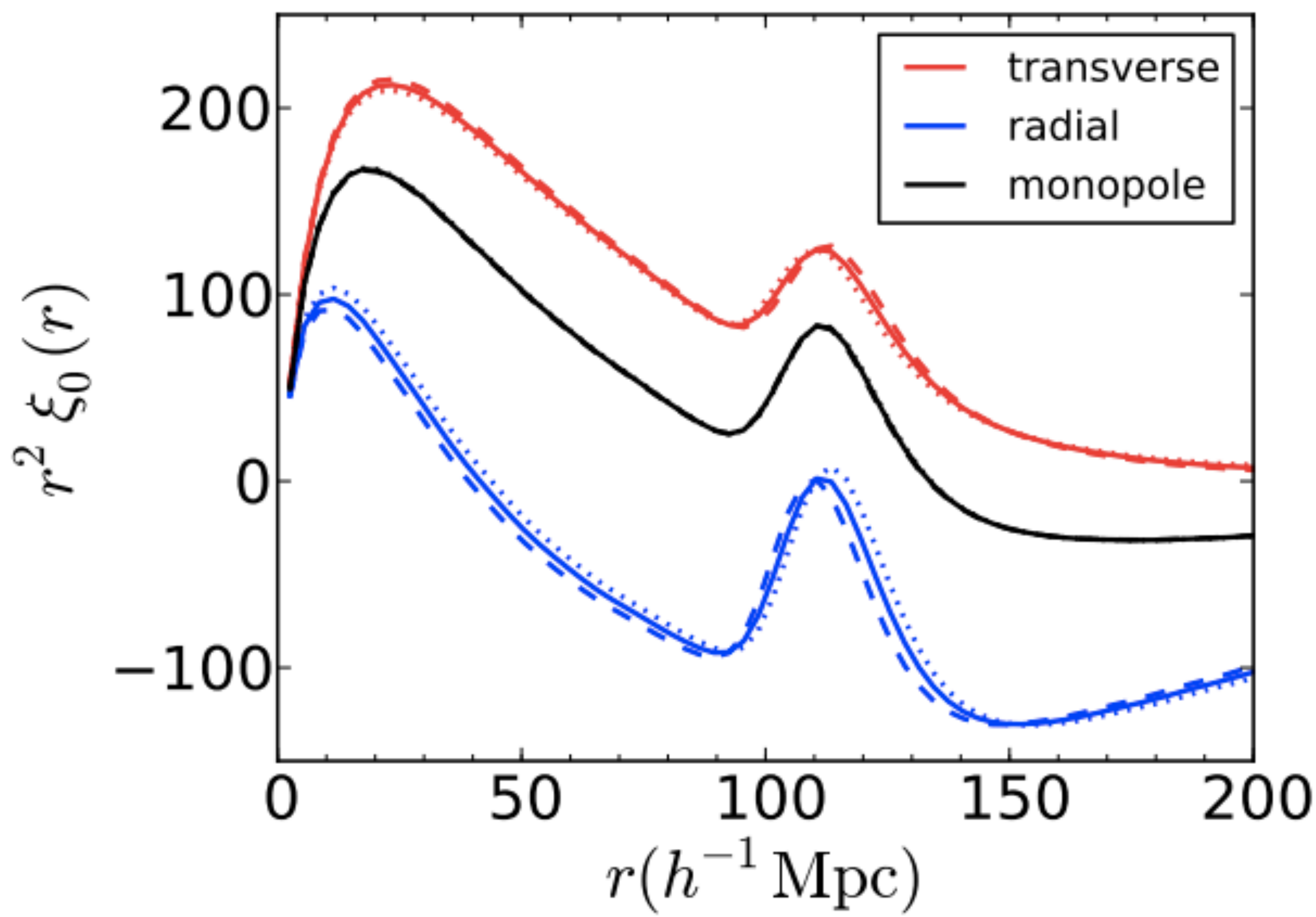


2% warp
5% warp
10% warp

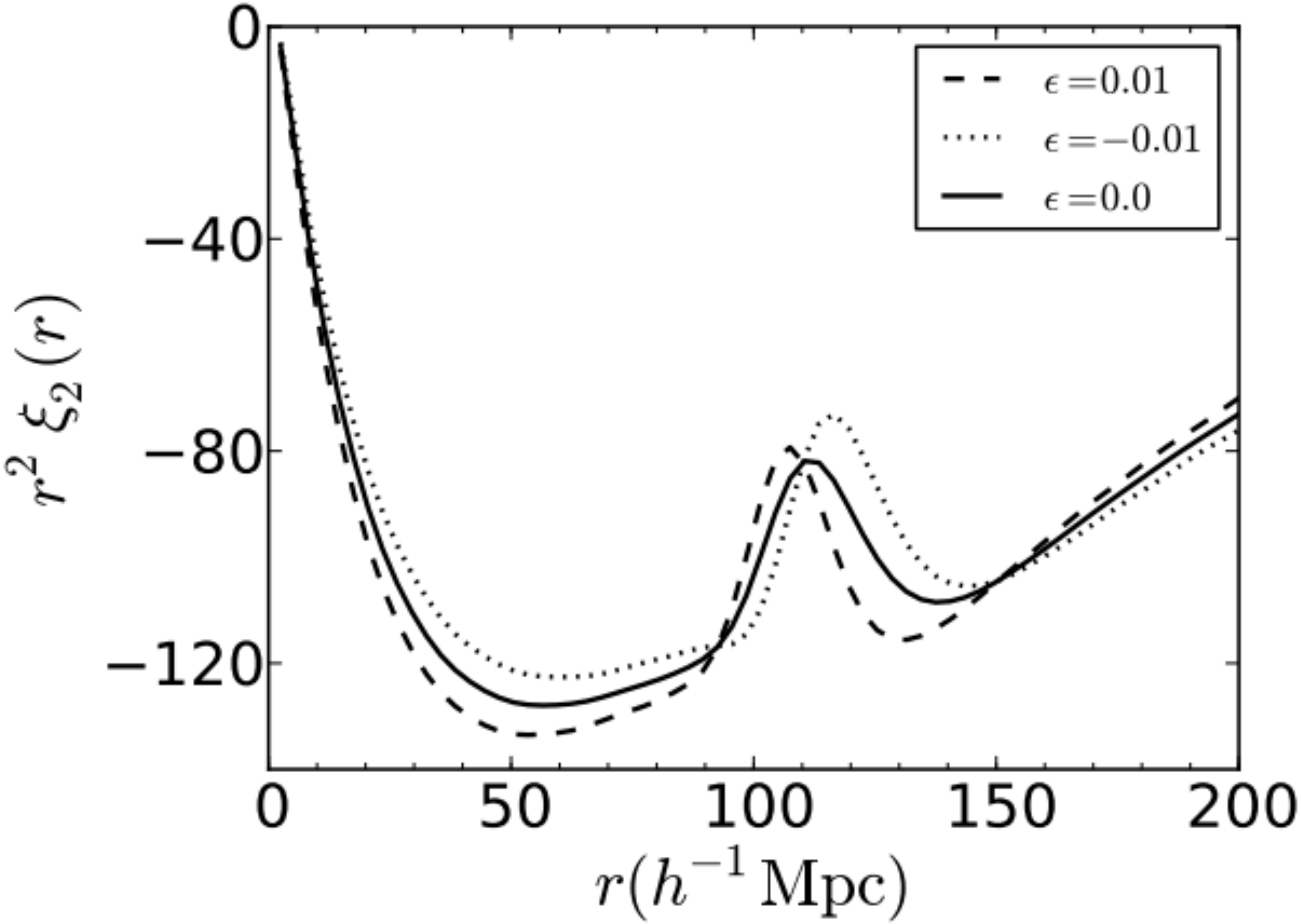
Effects on the correlation function



Effects on the correlation function

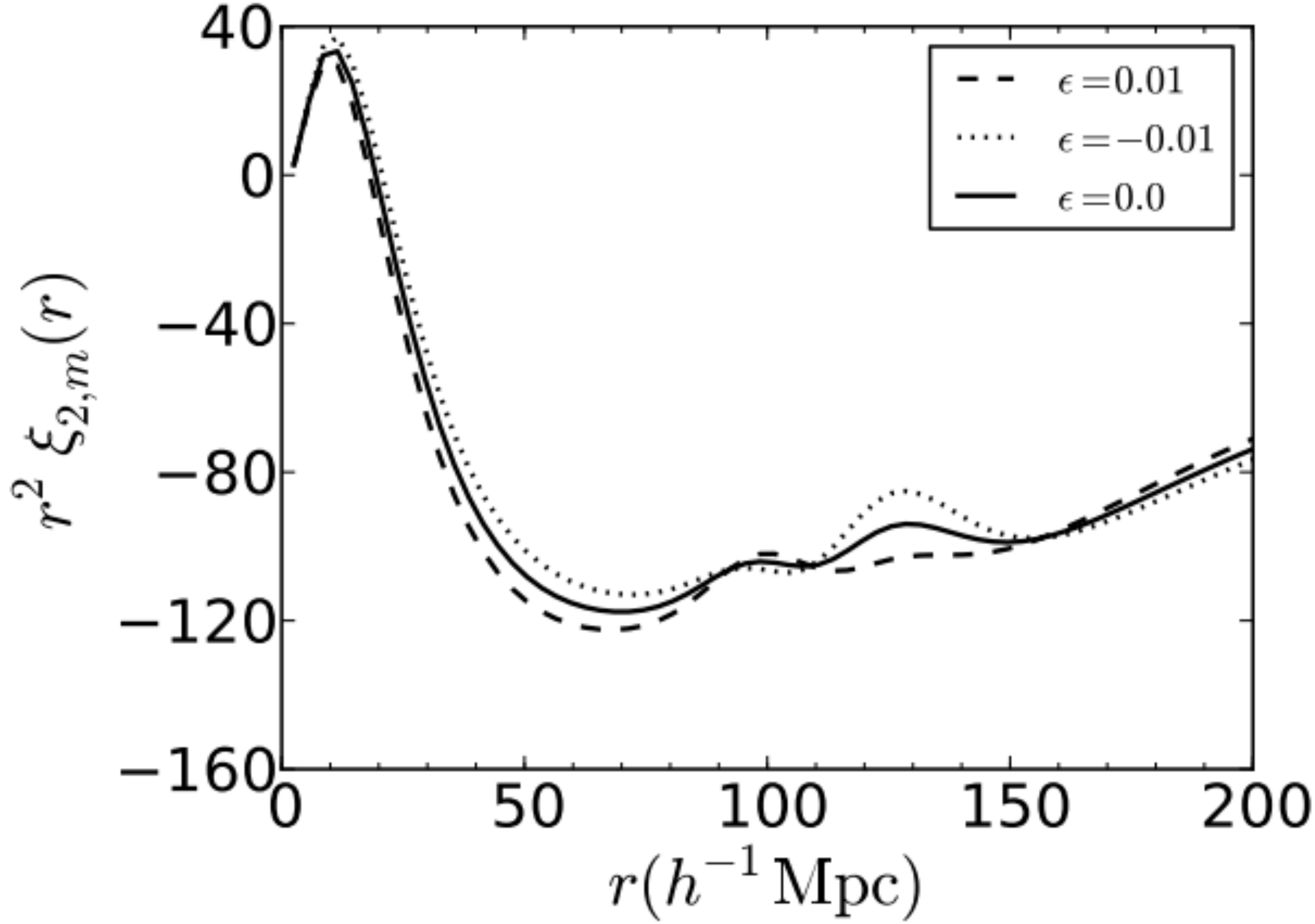


Effects on the correlation function



Xu et al, 2012

Including nonlinearities and RSD



Xu et al, 2012

Final comments

- This is not a unique treatment; can be done many ways
- The scaling with alpha and Dv depends on a survey where perpendicular and parallel separations are in the normal proportion. (eg. not true for Lyman-alpha measurements)

Measuring BAO : More comments

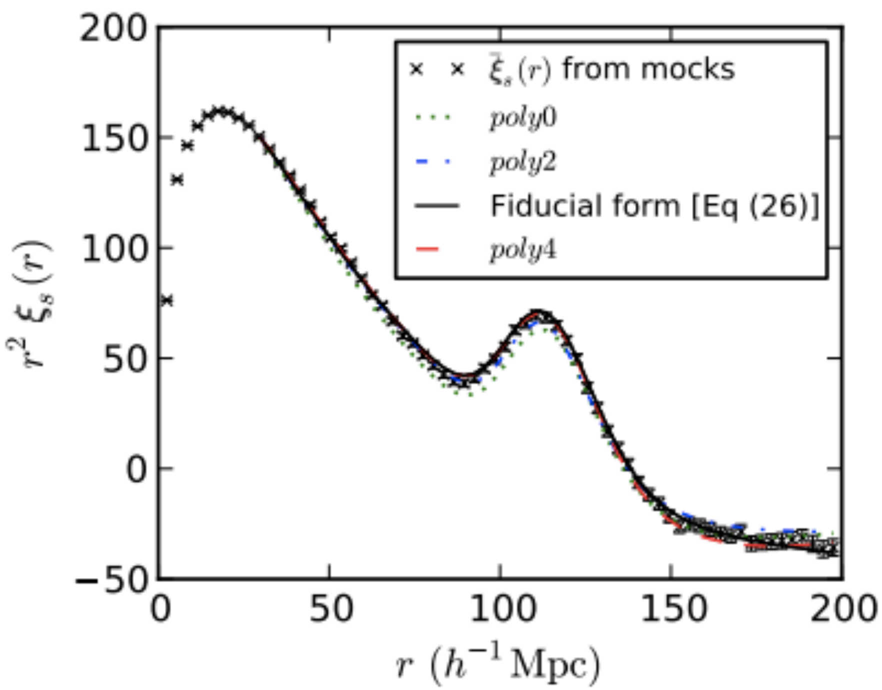
- In principle, any point in the correlation function may be used as a standard ruler
  - Except if the correlation function is a pure power law
  - This can be exploited to great effect eg. if you have a model for redshift space distortions
- BAO are special
  - A 3D feature in the matter distribution
  - Difficult to mimic
  - Broadband features may be affected by variations in cosmology, survey systematics etc.
- BAO measurements marginalize out shape information
  - Loss in information, gain in robustness

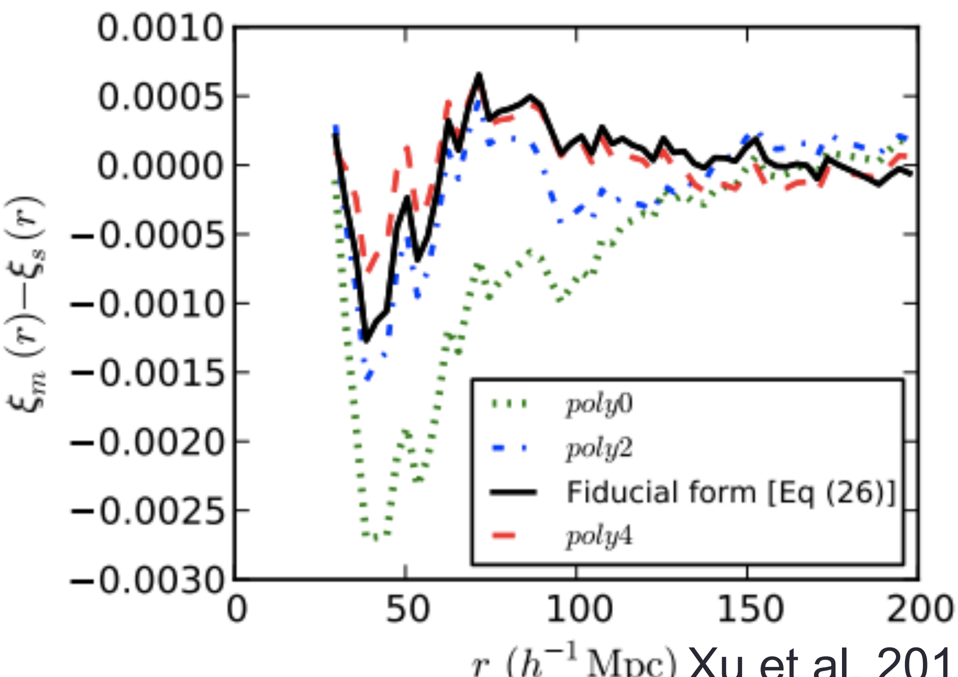
Marginalizing broadband

$$\xi^{fit}(r) = B^2 \xi_m(\alpha r) + A(r)$$

where

$$A(r) = \frac{a_1}{r^2} + \frac{a_2}{r} + a_3.$$





Xu et al, 2012

Building a template

Redshift space

$$P_t(k,\mu) = (1+\beta\mu^2)^2 F(k,\mu,\Sigma_s) P_{\rm dw}(k,\mu)$$

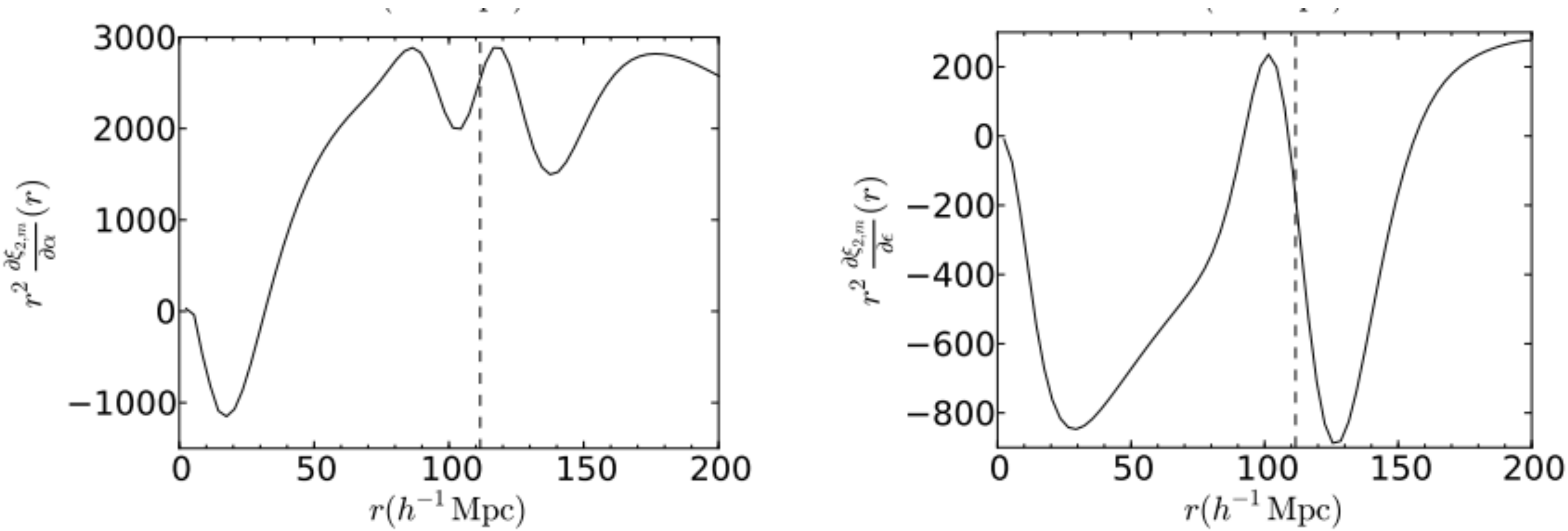
FoG

$$F(k,\mu,\Sigma_s) = \frac{1}{(1+k^2\mu^2\Sigma_s^2)^2}$$

Nonlinear evolution

$$\begin{aligned} P_{\rm dw}(k,\mu) &= [P_{\rm lin}(k) - P_{\rm nw}(k)] \\ &\quad \cdot \exp\left[-\frac{k^2\mu^2\Sigma_\parallel^2 + k^2(1-\mu^2)\Sigma_\perp^2}{2}\right] + P_{\rm nw}(k) \end{aligned}$$

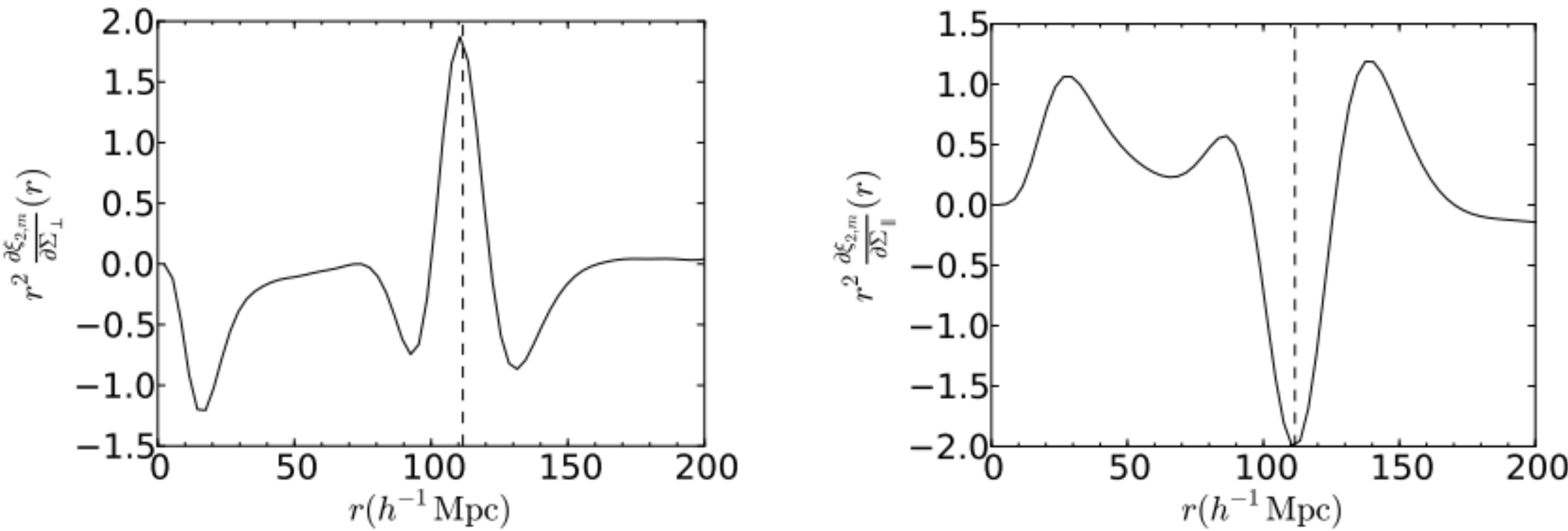
Degeneracies?



Note that alpha and epsilon look like shifts of the correlation function (see previous slide)

Xu et al, 2012

Degeneracies?



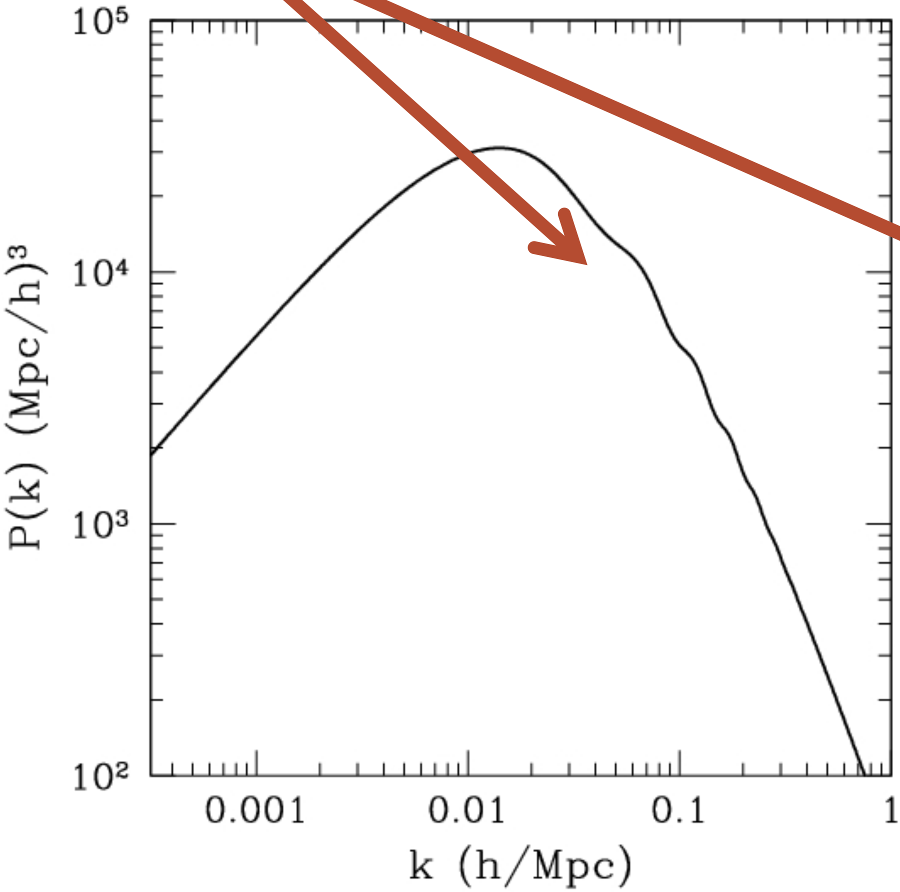
Note the different shape

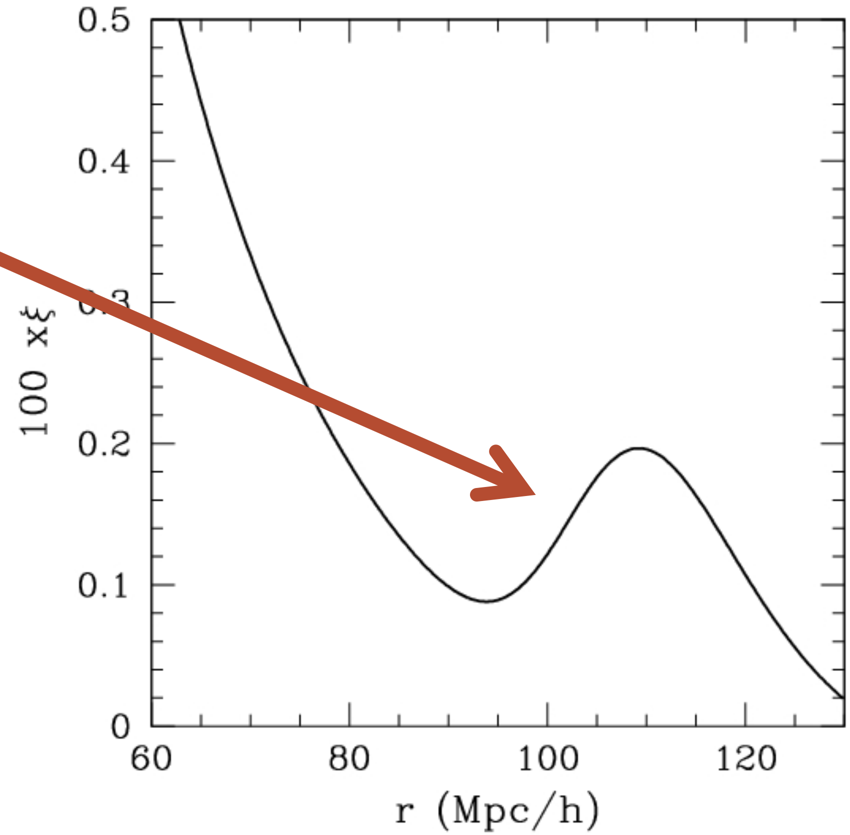
Xu et al, 2012

Nonlinearities : An apparent contradiction

- In configuration space : BAO feature ~ 100 Mpc/h : much larger than the nonlinear scale
- In Fourier space : BAO wiggles out to k~0.3-0.4 h/Mpc : these scales are strongly affected.
- How do you reconcile these pictures?

Ruler measured in galaxy maps





Wiggles in the power spectrum

Bump in the correlation function

Nonlinearities : An apparent contradiction

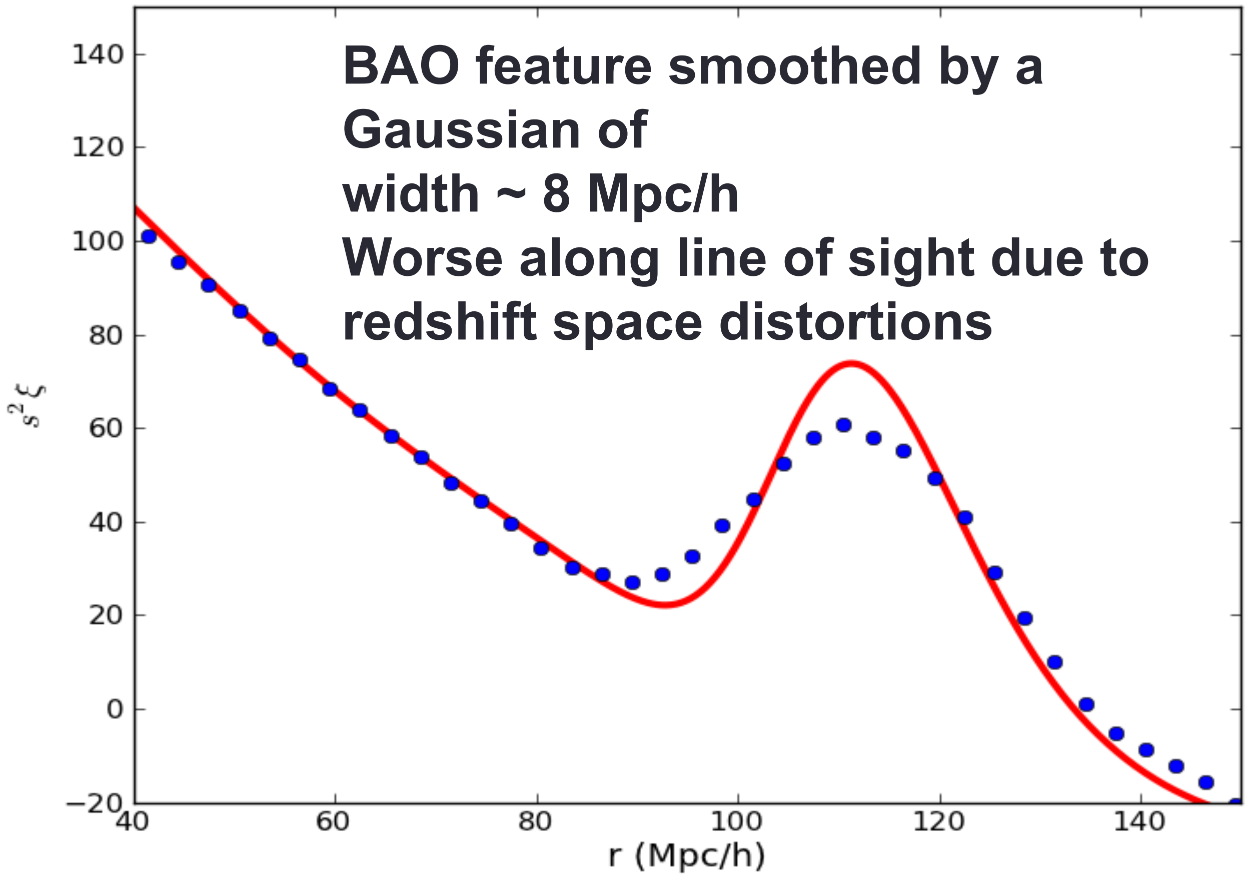
- In configuration space : BAO feature ~ 100 Mpc/h : much larger than the nonlinear scale
- In Fourier space : BAO wiggles out to k~0.3-0.4 h/Mpc : these scales are strongly affected.
- BAO scale encoded in differences between peaks and troughs : k ~ 0.03 h/Mpc
- $P(k) \sim j_0(k\, r_s)$
- Nonlinearities in P(k) are smooth on these scales (eg. Schulz and White 06)

See discussion in Eisenstein, Seo, White 2007

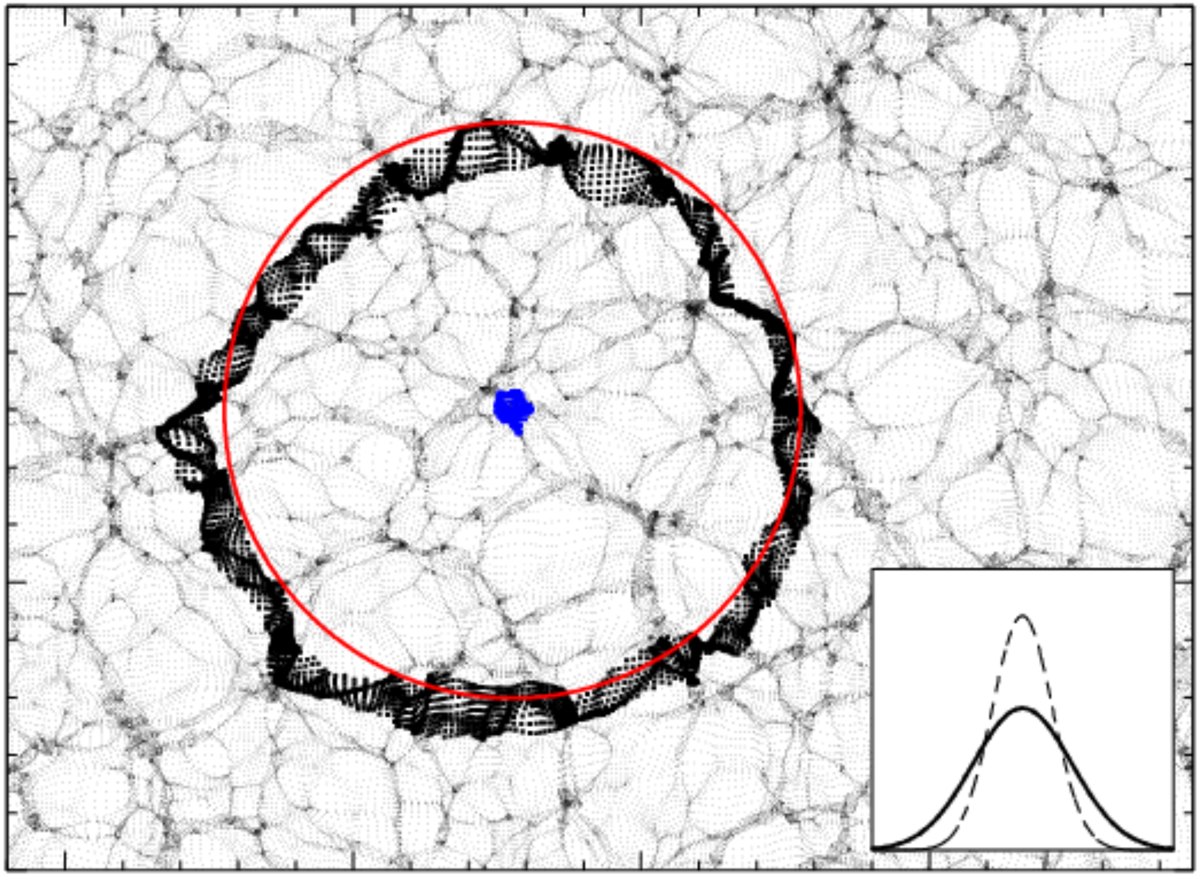
Damping and Shifts

- There are many treatments of this : Smith, Scoccimarro, Sheth 07, Eisenstein, Seo and White 07 (ESW07), Crocce & Scoccimarro 08, Matsubara 08 (M08), Padmanabhan and White 08 (P08), Sherwin and Zaldarriaga (120 and others.
- We will follow ESW07, M08, P08 in what follows, developing techniques we will use in later sections

The effect


BAO feature smoothed by a
Gaussian of
width ~ 8 Mpc/h
Worse along line of sight due to
redshift space distortions
140
120
100
80
60
40
20
0
−20
40
60
80
100
120
140
r (Mpc/h)
$s^2\xi$

The cartoon



Lagrangian PT : I

- Compute the pairwise displacement of pairs separated by ~100 Mpc
- Work in a Lagrangian framework

$$\boldsymbol{x}(\boldsymbol{q}, t) = \boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q}, t),$$

$$\frac{d^2\boldsymbol{\Psi}}{dt^2} + 2H\frac{d\boldsymbol{\Psi}}{dt} = -\boldsymbol{\nabla}_x \phi[\boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q})],$$

- The displacements can be expanded as a power series with the lowest order term being the Zeldovich approximation

$$\boldsymbol{\Psi} = \boldsymbol{\Psi}^{(1)} + \boldsymbol{\Psi}^{(2)} + \boldsymbol{\Psi}^{(3)} + \cdots,$$

Matsubara 08

LPT : II

$$\tilde{\boldsymbol{\Psi}}^{(n)}(\boldsymbol{p}) = \frac{iD^n}{n!}\int \frac{d^3p_1}{(2\pi)^3}\cdots\frac{d^3p_n}{(2\pi)^3}\delta^3\Big(\sum_{j=1}^{n}\boldsymbol{p}_j - \boldsymbol{p}\Big)$$
$$\times \boldsymbol{L}^{(n)}(\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n)\delta_0(\boldsymbol{p}_1)\cdots\delta_0(\boldsymbol{p}_n),$$

$$\boldsymbol{L}^{(1)}(\boldsymbol{p}_1) = \frac{\boldsymbol{k}}{k^2},$$

Eulerian perturbation theory has a very similar structure, but with different kernels; we will see this later.

Matsubara 08

Eisenstein, Seo, White 2007

Pairwise displacements

- Now just compute the pairwise displacements between points separated in Lagrangian space

$$\vec{u}_{12} = \int \frac{d\vec{k}}{(2\pi)^3} \delta_{\vec{k}} \frac{\vec{k}}{ik^2} \left[e^{i\vec{k}\cdot\vec{r}_1} - e^{i\vec{k}\cdot\vec{r}_2} \right]$$

This quantity must have mean 0, so compute the dispersion – along and transverse to the displacement vector

$$\left\langle u_{12,\parallel}^2 \right\rangle = \int \frac{d\vec{k}}{(2\pi)^3} P(k) \left(\frac{\vec{k}\cdot\vec{r}_{12}}{k^2 r_{12}} \right)^2 \left| e^{i\vec{k}\cdot\vec{r}_{12}} - 1 \right|^2 = r_{12}^2 \int \frac{k^2\,dk}{2\pi^2} P(k) f_\parallel(kr_{12})$$

$$f_\parallel(x) = \frac{2}{x^2}\left(\frac{1}{3} - \frac{\sin x}{x} - \frac{2\cos x}{x^2} + \frac{2\sin x}{x^3} \right) \qquad f_\perp(x) = \frac{2}{x^2}\left(\frac{1}{3} - \frac{\sin x}{x^3} + \frac{\cos x}{x^2} \right)$$

Limits

- Start by considering the k->0 limit

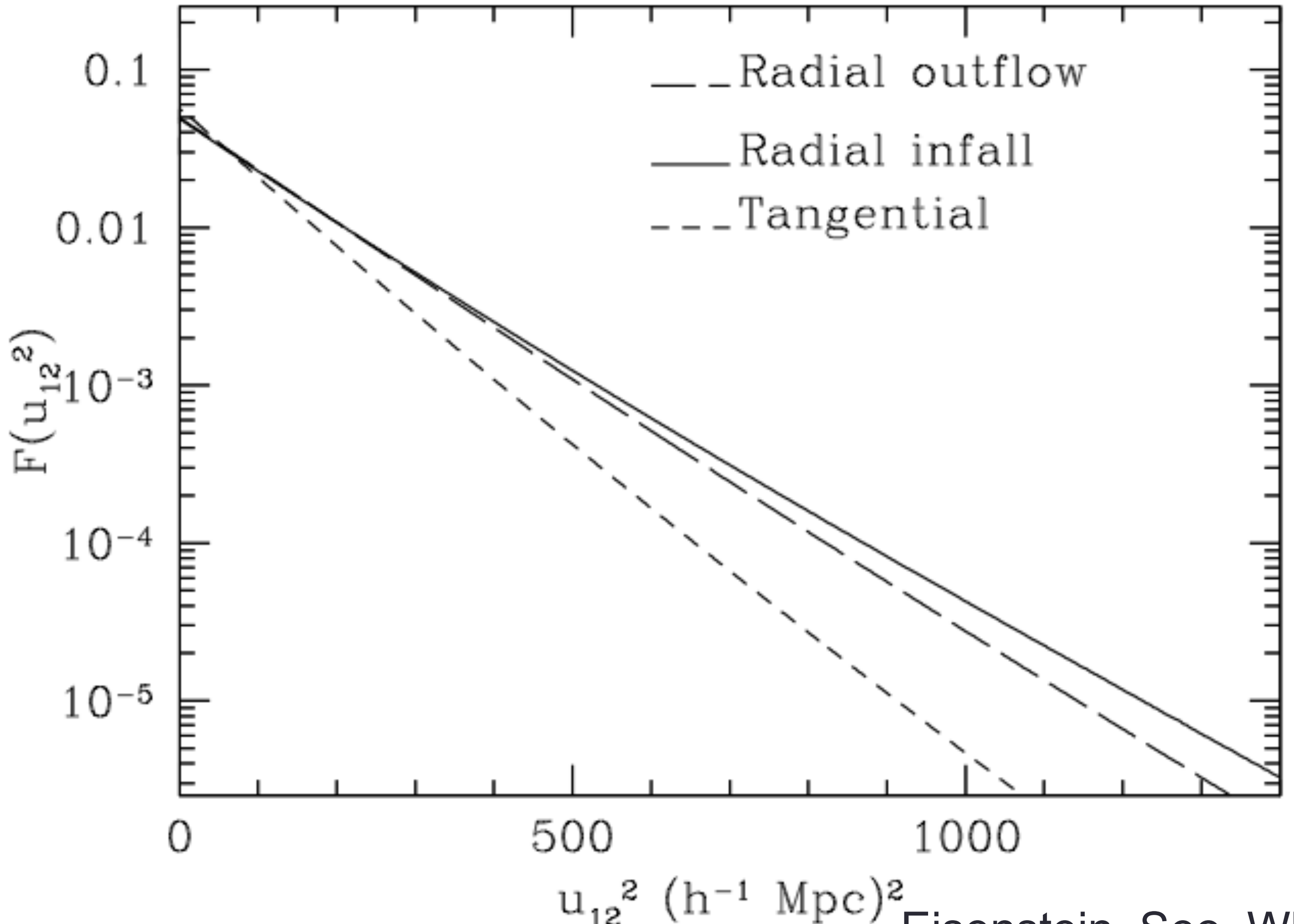
$$f_{||} \rightarrow 1/5 - x^2/84 \text{ as } x \rightarrow 0$$

$$f_{\perp}(x) \quad 1/15 - x^2/420 \text{ as } x \rightarrow 0$$

- Variance goes to zero in this limit
- Physically corresponds to moving pairs by the same long wavelength mode – no relative displacement (this physical condition will be explicitly violated in the next discussion).
- Very little variance coming from scales k < 0.02 h/Mpc
- ~50% of the variance coming from scales k < ~0.05-0.08 h/Mpc – large scales (confirming our basic picture)

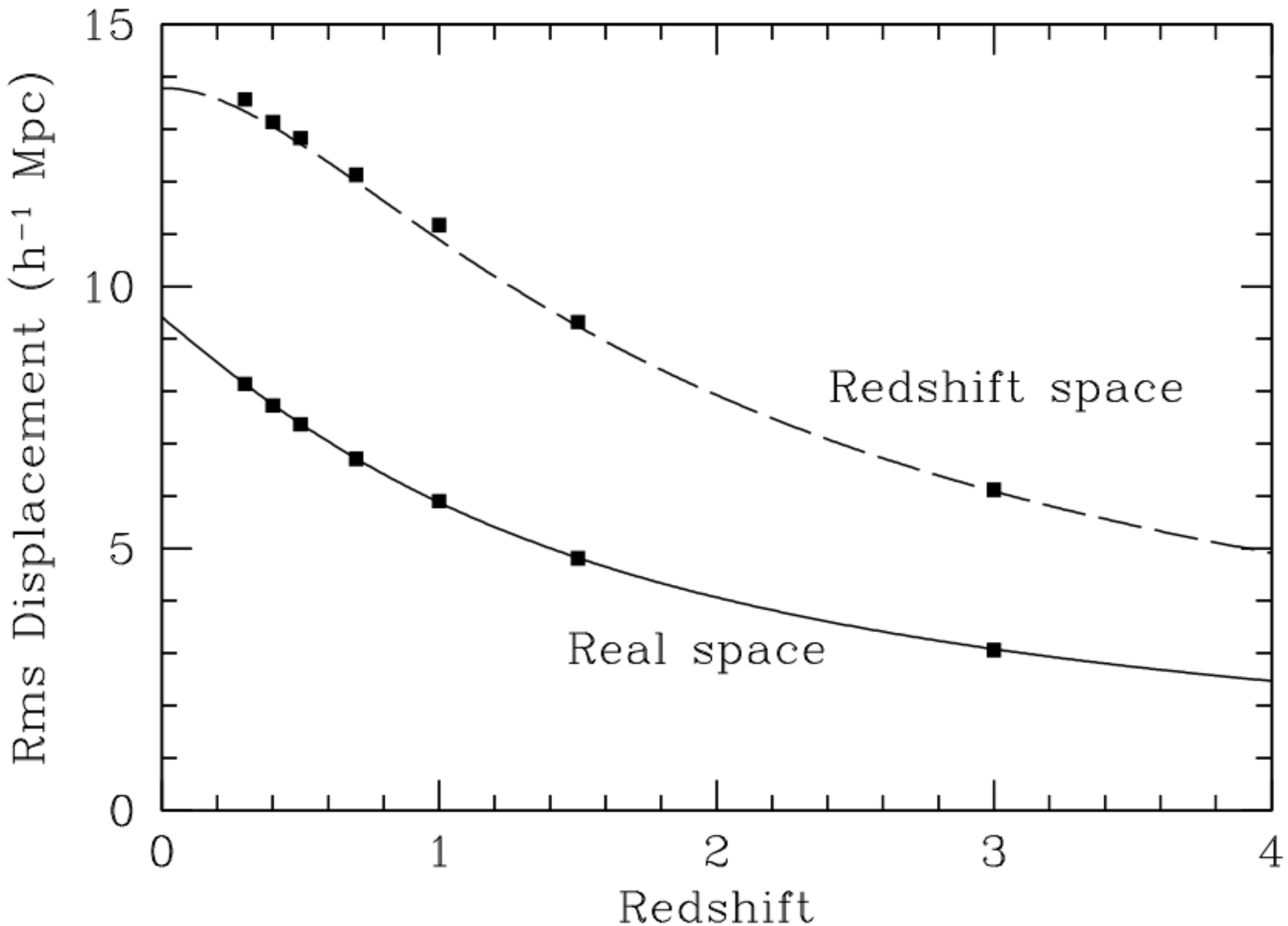
Simulations

Dispersions are well approximated by a Gaussian



Eisenstein, Seo, White 2007

Simulations vs Theory



Eisenstein, Seo, White 2007

Damping, a different way

- Continue our discussion of LPT

$$\delta(\boldsymbol{x}) = \int d^3q\,\delta^3[\boldsymbol{x} - \boldsymbol{q} - \boldsymbol{\Psi}(\boldsymbol{q})] - 1,$$

$$\tilde{\delta}(\boldsymbol{k}) = \int d^3x e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\delta(\boldsymbol{x})$$

$$\langle\tilde{\delta}(\boldsymbol{k})\tilde{\delta}(\boldsymbol{k}')\rangle = (2\pi)^3\delta^3(\boldsymbol{k} + \boldsymbol{k}')P(\boldsymbol{k}),$$

$$P(\boldsymbol{k}) = \int d^3q e^{-i\boldsymbol{k}\cdot\boldsymbol{q}}(\langle e^{-i\boldsymbol{k}\cdot[\boldsymbol{\Psi}(\boldsymbol{q}_1)-\boldsymbol{\Psi}(\boldsymbol{q}_2)]}\rangle - 1),$$

The cumulant expansion

- Normally, expand the exponential and build up a perturbation series.
- Matsubara 08 proposed a alternative : use the cumulant expansion

$$\langle e^{-iX} \rangle = \exp\left[\sum_{N=1}^{\infty} \frac{(-i)^N}{N!} \langle X^N \rangle_c \right],$$

- Two classes of terms : terms evaluated at 1 point and terms evaluated at 2 points

$$\langle \{ \boldsymbol{k} \cdot [\boldsymbol{\Psi}(\boldsymbol{q}_1) - \boldsymbol{\Psi}(\boldsymbol{q}_2)] \}^N \rangle_c = [1 + (-1)^N] \langle [\boldsymbol{k} \cdot \Psi(0)]^N \rangle_c + \sum_{j=1}^{N-1} (-1)^{N-j} \binom{N}{j} \times \langle [\boldsymbol{k} \cdot \Psi(\boldsymbol{q}_1)]^j [\boldsymbol{k} \cdot \Psi(\boldsymbol{q}_2)]^{N-j} \rangle_c,$$

Only N>1 survive

Damping

- Keep the 1 point terms in the exponential, expand the 2 point terms

$$P(k) = \exp\left[-\frac{k^2}{6\pi^2}\int dp P_{\mathrm{L}}(p)\right]\left[P_{\mathrm{L}}(k) + P_{\mathrm{SPT}}^{1\text{-loop}}(k) + \frac{k^2}{6\pi^2}P_{\mathrm{L}}(k)\int dp P_{\mathrm{L}}(p)\right],$$

- Compare results to previous estimate

Damping in simulations

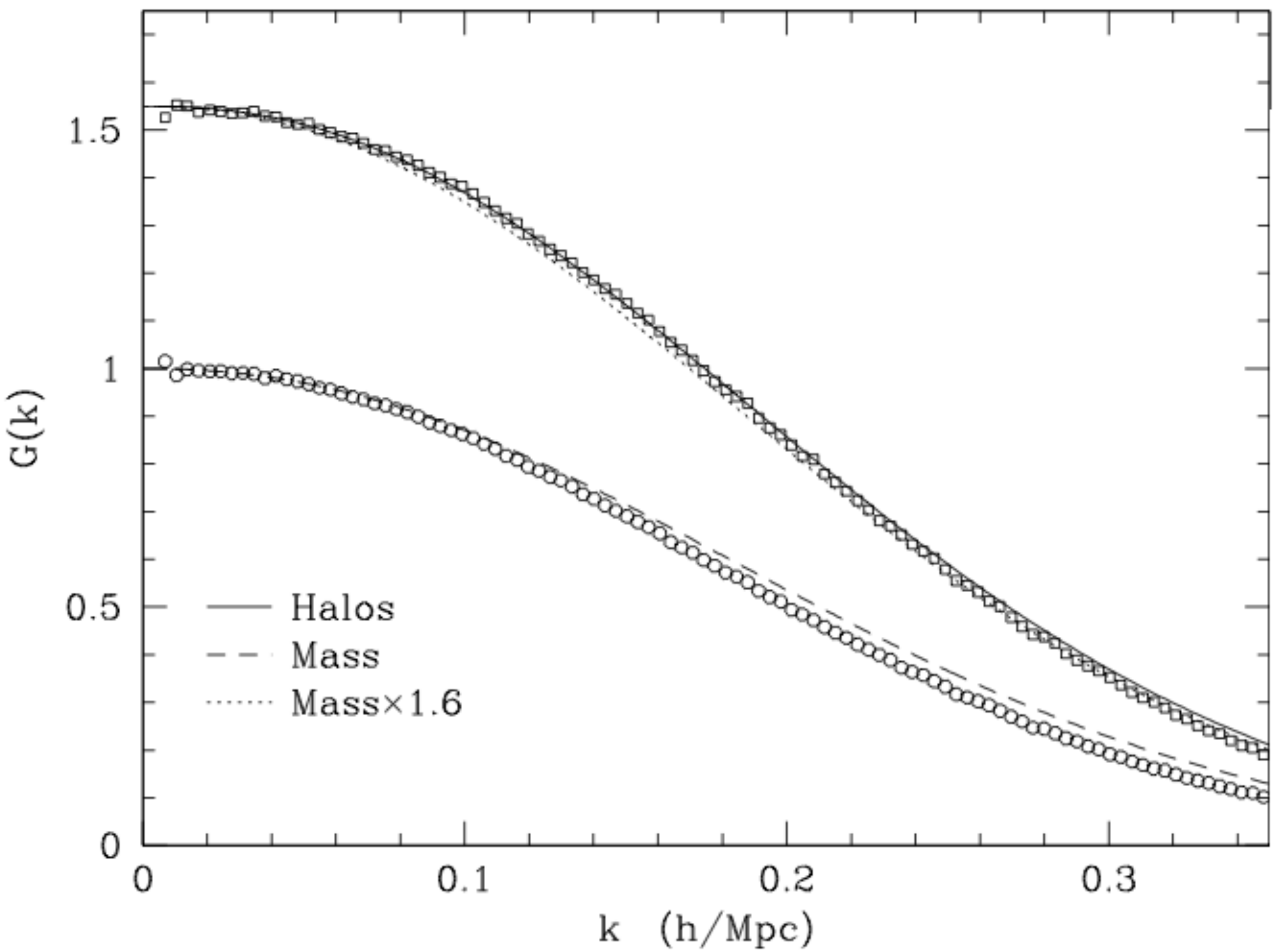
- Cross correlate with the initial density field

$$G_f(k) \equiv \frac{\left\langle \delta_L(k)\delta_f^\star(k) \right\rangle}{P_L}$$

$$G_f(k) \simeq e^{-(k\Sigma)^2/4} + \cdots$$

$$\Sigma^2 = \frac{1}{3\pi^2}\int dq\, P_L(p)$$

Damping in simulations



Noh, White & NP 2009

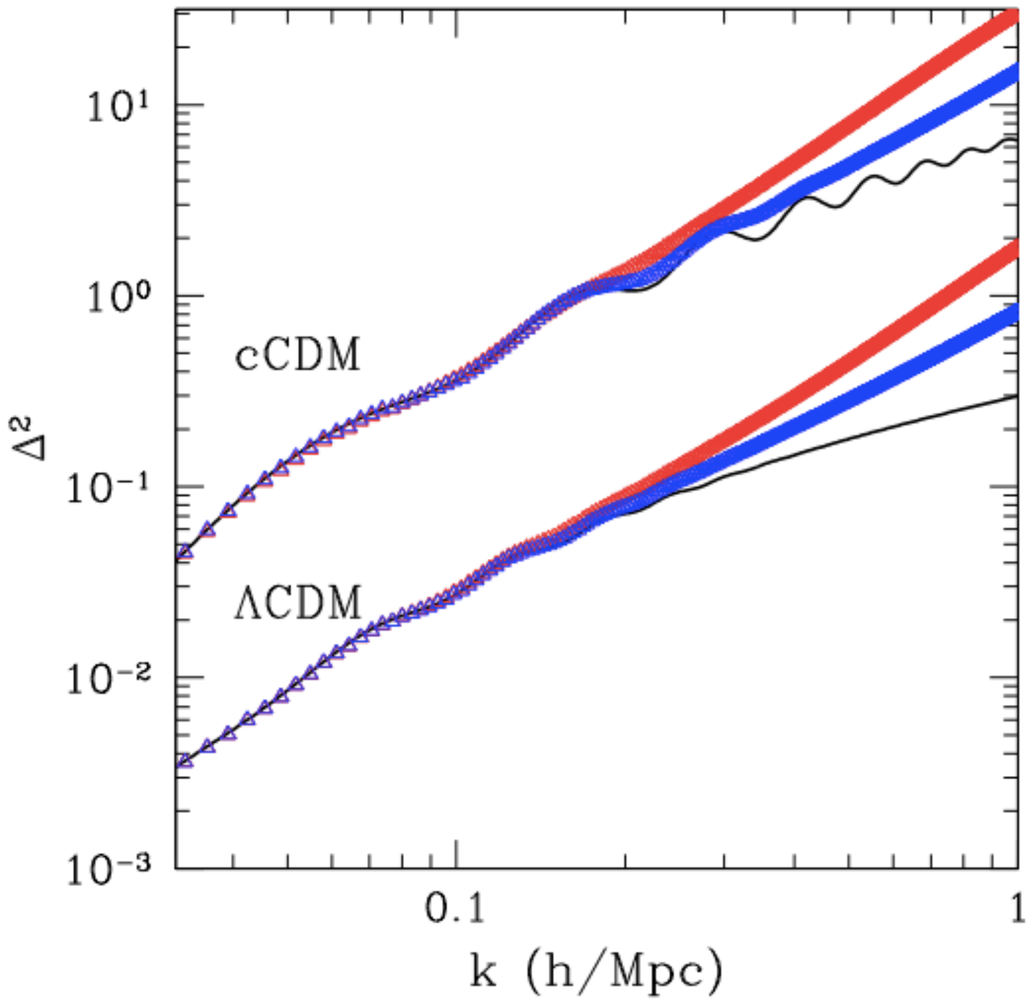
Shifts

- Is the BAO ruler actually standard?
  - What would shifts look like?

$$P_L(k/\alpha) \simeq P_L(k) - (\alpha - 1)\,\frac{dP_L}{d\ln k} + \cdots$$

- Treat simulations and theory as data : remove broadband shifts
- No shifts => alpha=1

Numerical experiments



Back to perturbative expansions

$$\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \cdots$$

The Fn kernels can be “simply” calculated by recursion relations (Jain and Bertschinger)

$$\delta^{(n)}(k) \;=\; \int \frac{d^3q_1 \ldots d^3q_n}{(2\pi)^{3n}}\,(2\pi)^3\delta_D\left(\sum q_i - k\right) \times\; F_n(\{q_i\},k)\,\delta_L(q_1)\ldots\delta_L(q_n)$$

Regroup terms

$$P_{NL} = \{P_{11} + P_{13} + P_{15} + \cdots\} + \{P_{22} + \cdots\}$$

P_{1n}

- If we focus on the lowest order term, this is a broad integral over k – expect to wash out oscillations and would not expect a shift

$$P_{1n}(k) \sim P_L(k) \int \prod_{k=1}^{(n-1)/2} \left[d^3 q_k P_L(q_k) \right] F_n(\cdots) \, .$$

P_{mn}, m,n>1

$$Q_n(k) = \frac{k^3}{4\pi^2}\int_0^\infty dr\, P_L(kr)\int_{-1}^{1} dx\, P_L(k\sqrt{y})\widetilde{Q}_n(r,x)$$

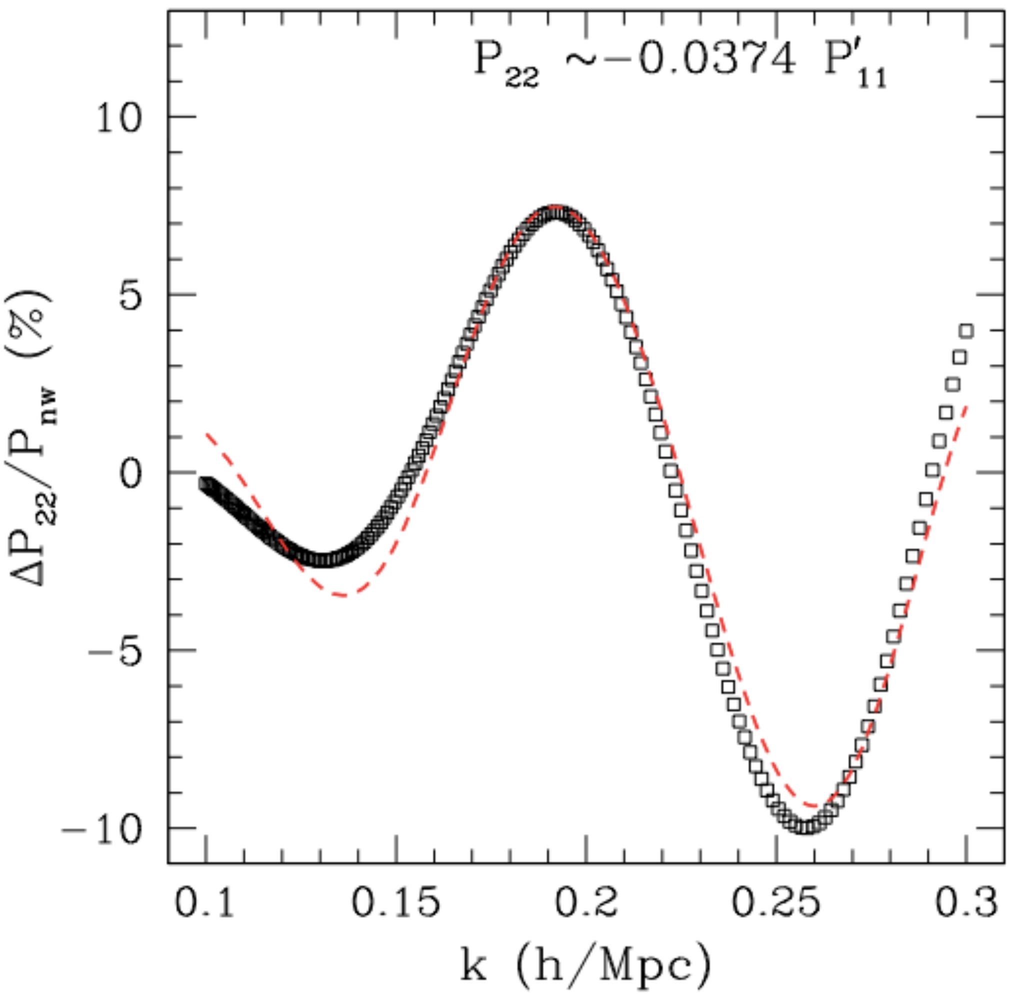
$$\begin{aligned}
&\widetilde{Q}_1 = \frac{r^2(1-x^2)^2}{y^2},\ \widetilde{Q}_2 = \frac{(1-x^2)rx(1-rx)}{y^2},\\
&\widetilde{Q}_3 = \frac{x^2(1-rx)^2}{y^2},\ \widetilde{Q}_4 = \frac{1-x^2}{y^2},\\
&\widetilde{Q}_5 = \frac{rx(1-x^2)}{y},\ \ \widetilde{Q}_6 = \frac{(1-3rx)(1-x^2)}{y},\\
&\widetilde{Q}_7 = \frac{x^2(1-rx)}{y},\ \ \widetilde{Q}_8 = \frac{r^2(1-x^2)}{y},\\
&\widetilde{Q}_9 = \frac{rx(1-rx)}{y},\ \ \widetilde{Q}_{10} = 1-x^2,\\
&\widetilde{Q}_{11} = x^2,\ \widetilde{Q}_{12} = rx,\ \widetilde{Q}_{13} = r^2
\end{aligned}$$

P(k) ~ sin(k)

Q ~ sin^2(k/2) ~ cos(k)

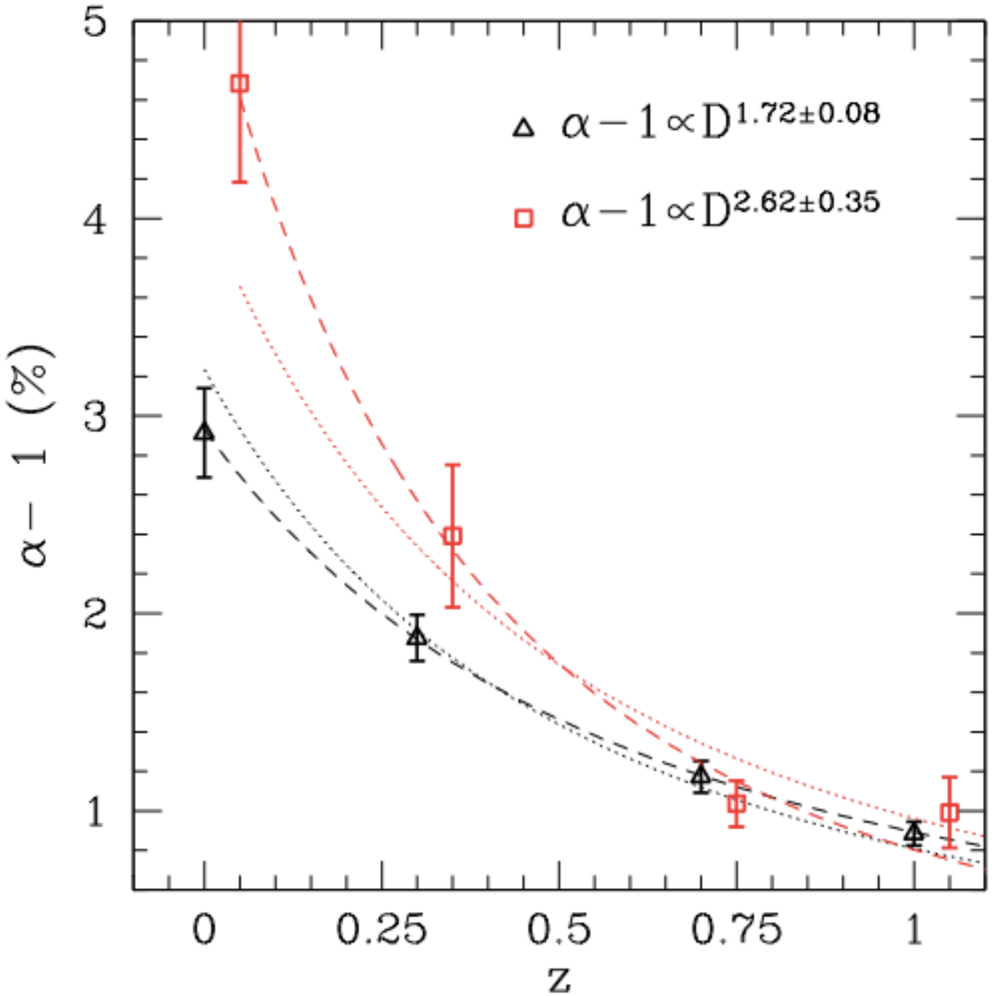
These will cause shifts

P_{22} (warning : not concordance)



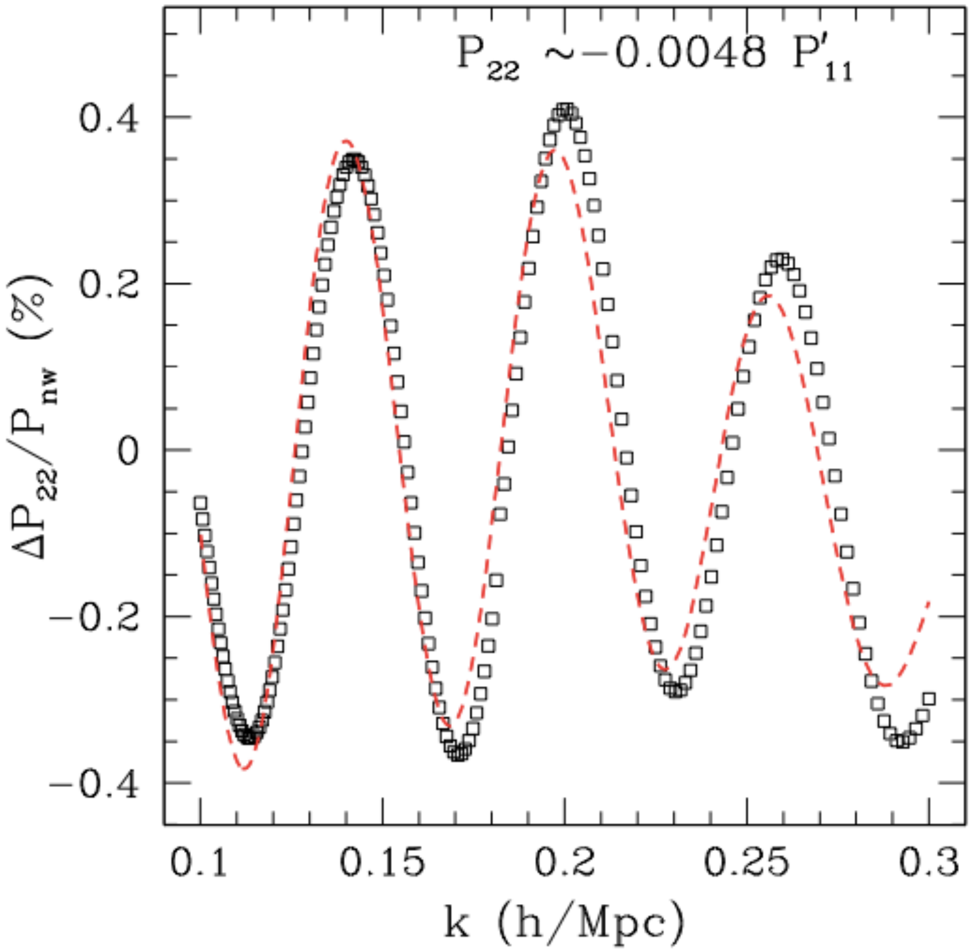
NP & White, 2009

Shifts (warning : non-standard)



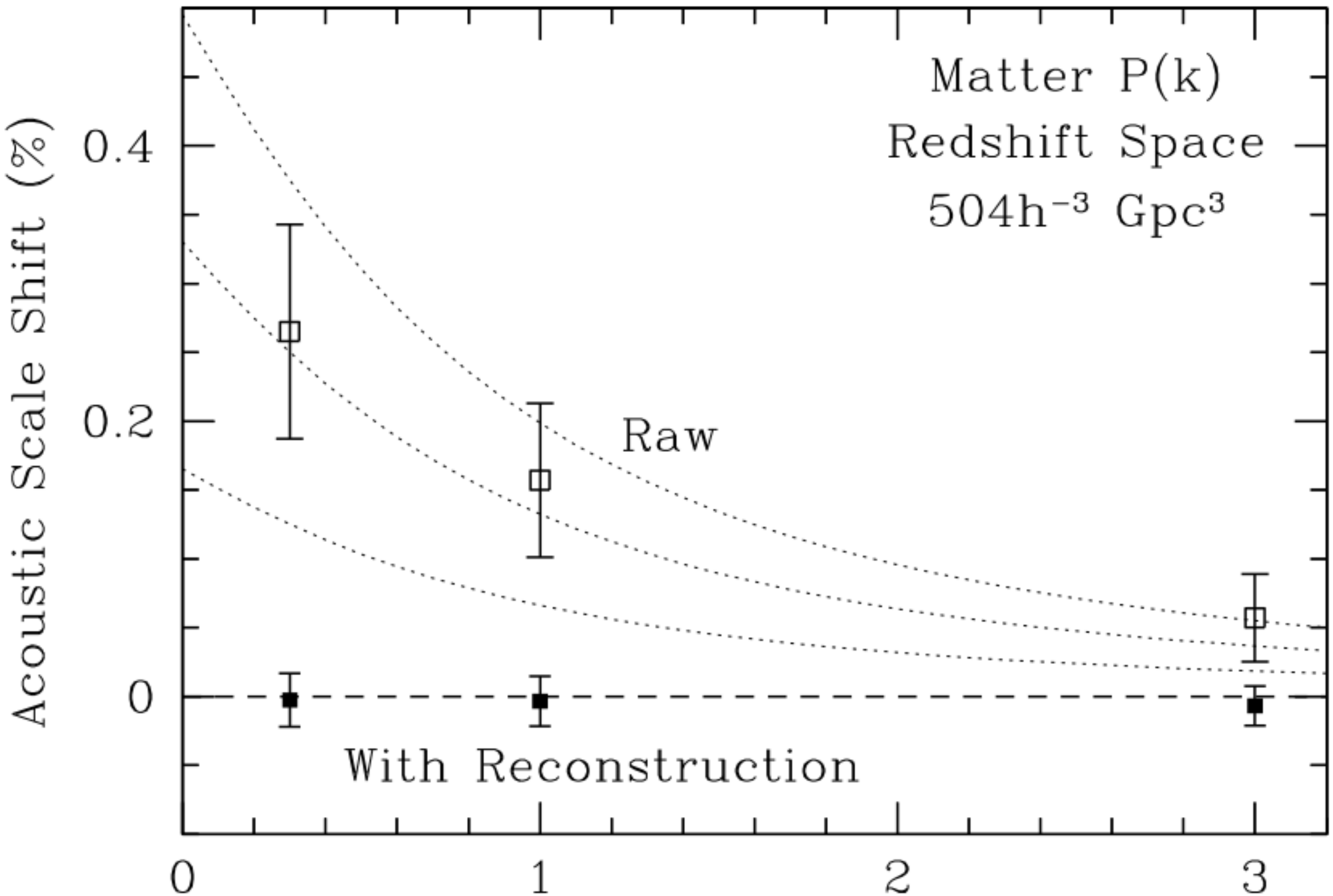
NP & White, 2009

Predicted shifts (matter)



NP & White, 2009

Shifts, in simulations



Seo et al, 2010

What about biased tracers?

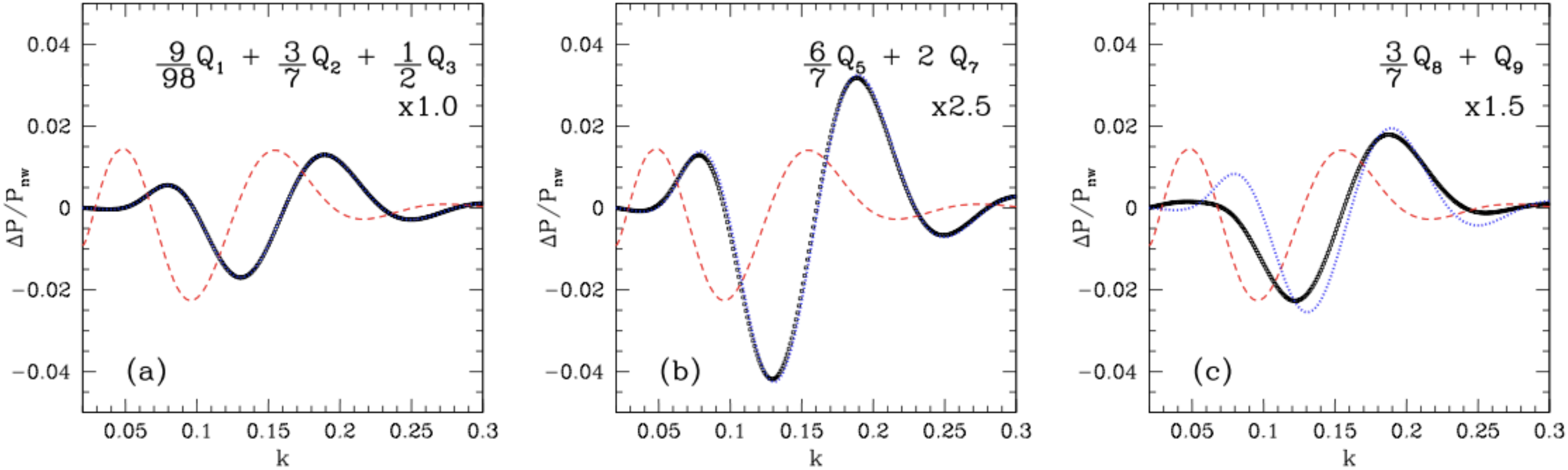
- Expand the galaxy density as a function of the matter density field

$$\delta_h = b_1^E \delta + \frac{b_2^E}{2!}\delta^2 + \cdots$$

- Working through the PT calculations, get different combinations of Qs

$$\begin{aligned} P_h &= \left(b_1^E\right)^2 (P_{11} + P_{22}) + b_1^E b_2^E \left(\frac{3}{7}Q_8 + Q_9\right) \\ &+ \frac{(b_2^E)^2}{2} Q_{13} + \cdots \end{aligned}$$

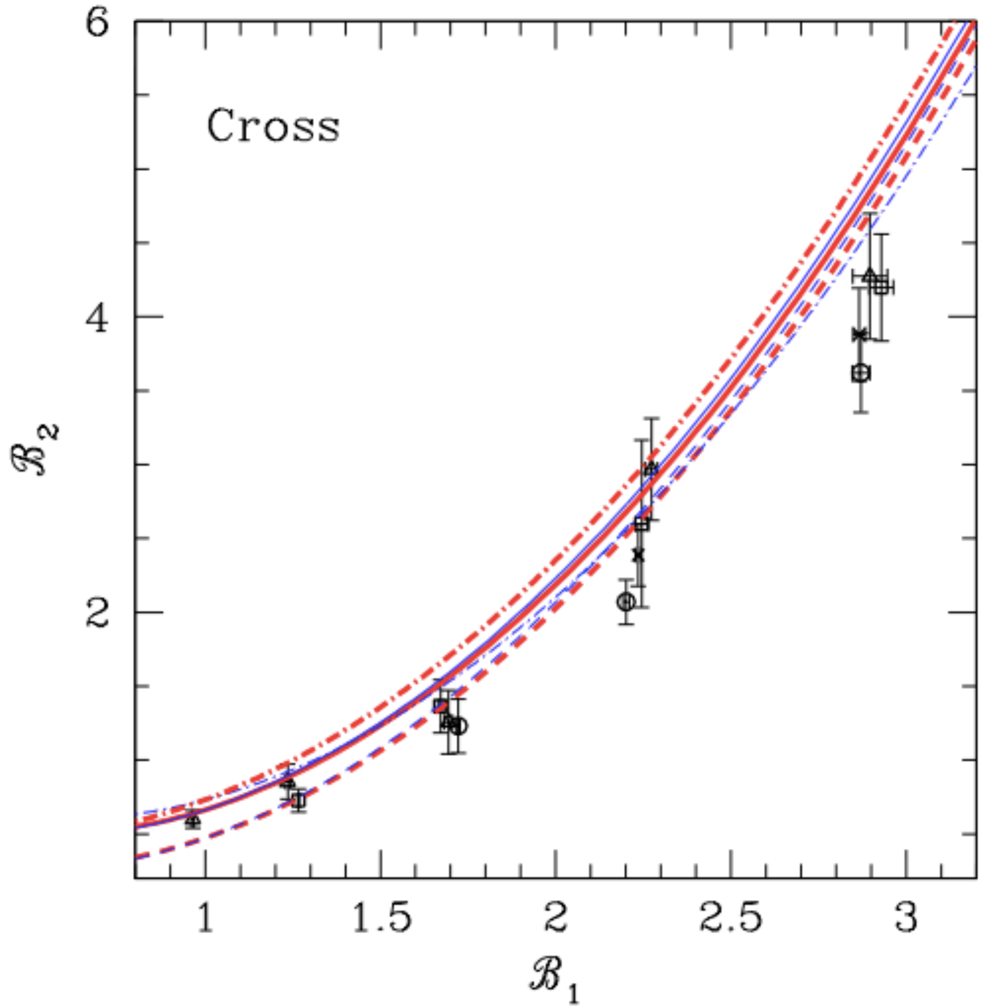
Look like shifts



$$P_h = \exp\left(-\frac{k^2\Sigma^2}{2}\right)\left[\mathcal{B}_1 P_L + \mathcal{B}_2 P_{22}\right].$$

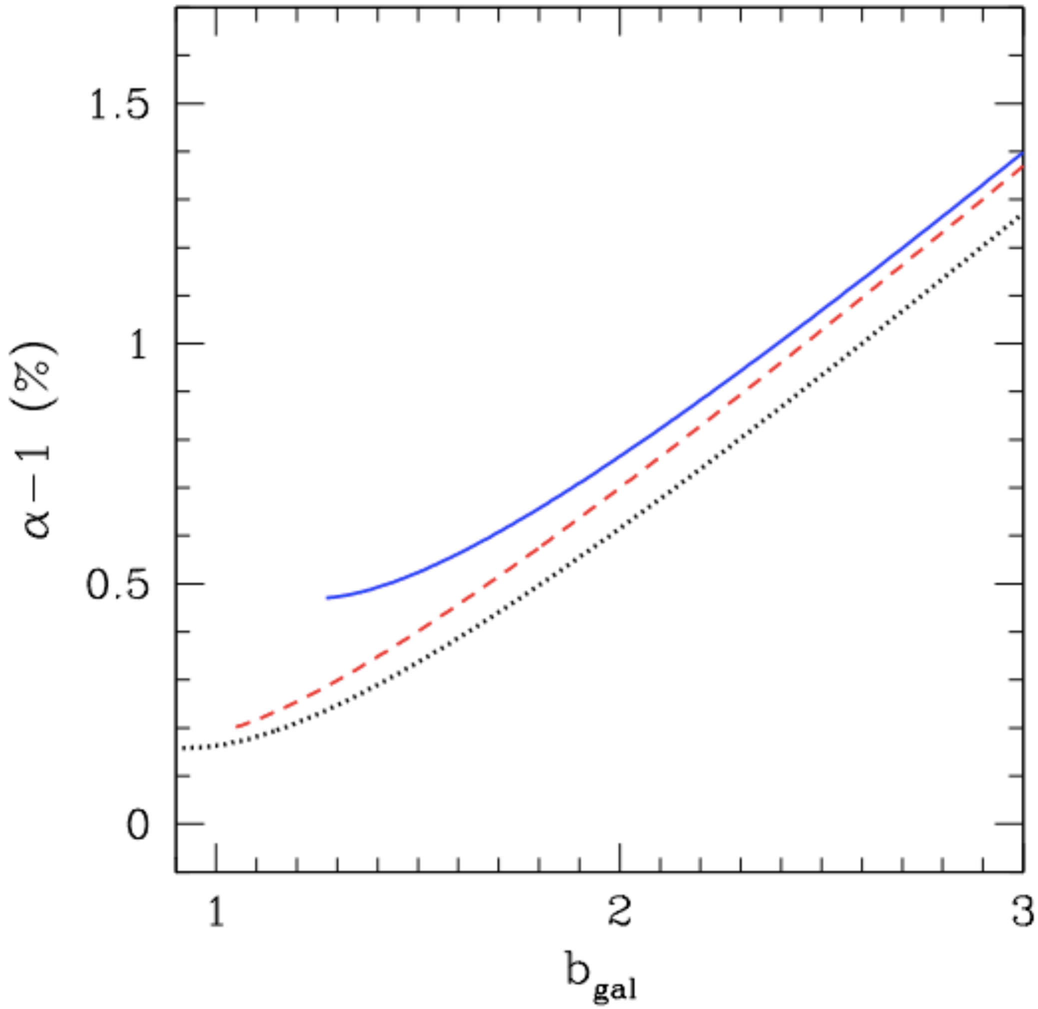
NP & White, 2009

We can predict these



NP & White, 2009

Estimates of shifts as a function of bias



- Note that we don't need these to (sub-) percent levels

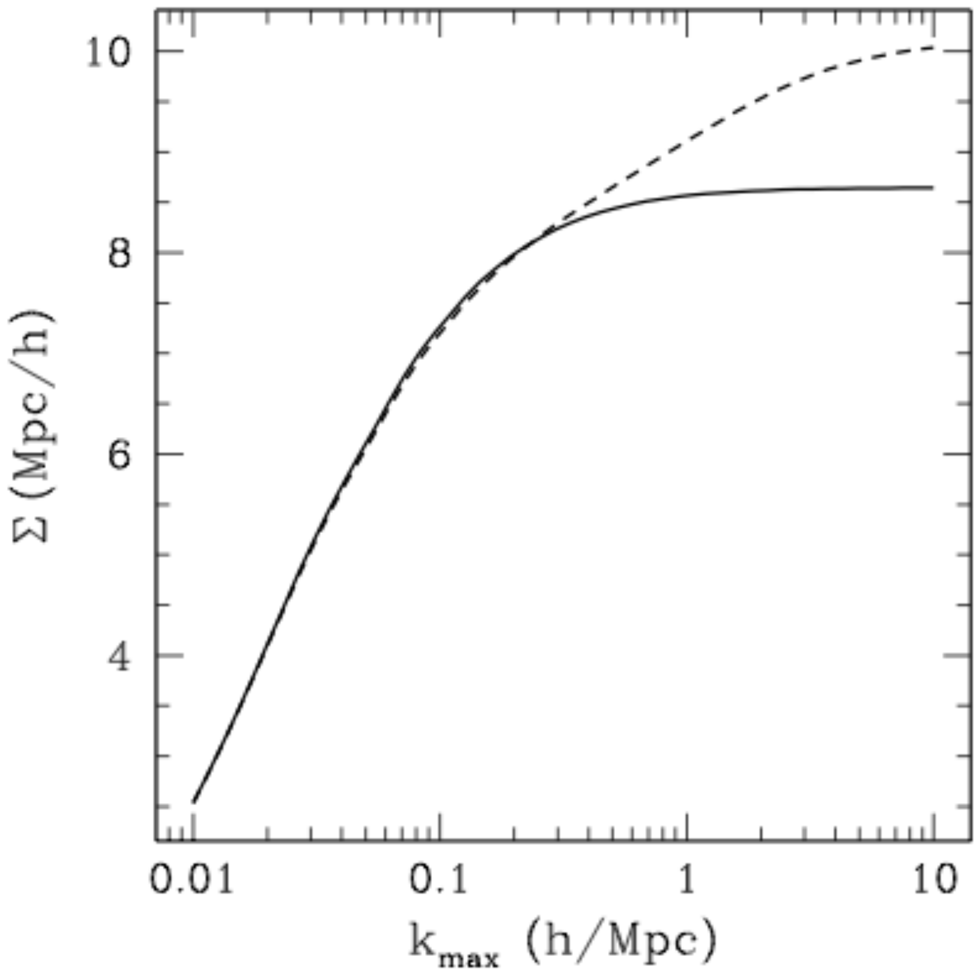
NP & White, 2009

Reconstruction : An algorithm

- Smooth the density field
- Estimate the Zel'dovich displacement
- Shift galaxies back by –displacement
- Shift randoms back by –displacement
- Reconstructed density field : galaxies – randoms
  - This is ideally suited to Lagrangian PT
  - Can we understand this algorithm?

Eisenstein et al, 2007

Damping comes from large scales



NP, White, Cohn 2009

Implement reconstruction in LPT

$$\delta^{(2)} = \int d^3q\, e^{-i\mathbf{k}\cdot\mathbf{q}} \left[-i\mathbf{k}\mathbf{\Psi}^{(2)} - \frac{(\mathbf{k}\cdot\mathbf{\Psi}^{(1)})^2}{2} \right]$$

$$\mathbf{s}(\mathbf{k}) \equiv -i\frac{\mathbf{k}}{k^2}\mathcal{S}(k)\delta(\mathbf{k})$$

$$\delta_{\rm recon}(\mathbf{k}) = \int d^3q e^{-i\mathbf{k}\cdot\mathbf{q}} e^{-i\mathbf{k}\cdot\mathbf{s}(\mathbf{q})} \left(e^{-i\mathbf{k}\cdot\mathbf{\Psi}(\mathbf{q})} - 1 \right)$$

$$\begin{aligned}
\delta^{(2)}_{\rm recon} &= \delta^{(2)} - \frac{1}{2}\int \frac{d^3k_1 d^3k_2}{(2\pi)^3}\delta^{(D)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}) \\
&\times \delta_l(\mathbf{k}_1)\delta_l(\mathbf{k}_2)\, \mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_1)\mathbf{k}\cdot\mathbf{L}^{(1)}(\mathbf{k}_2) \\
&\times [\mathcal{S}(\mathbf{k}_1)+\mathcal{S}(\mathbf{k}_2)] \ .
\end{aligned}$$

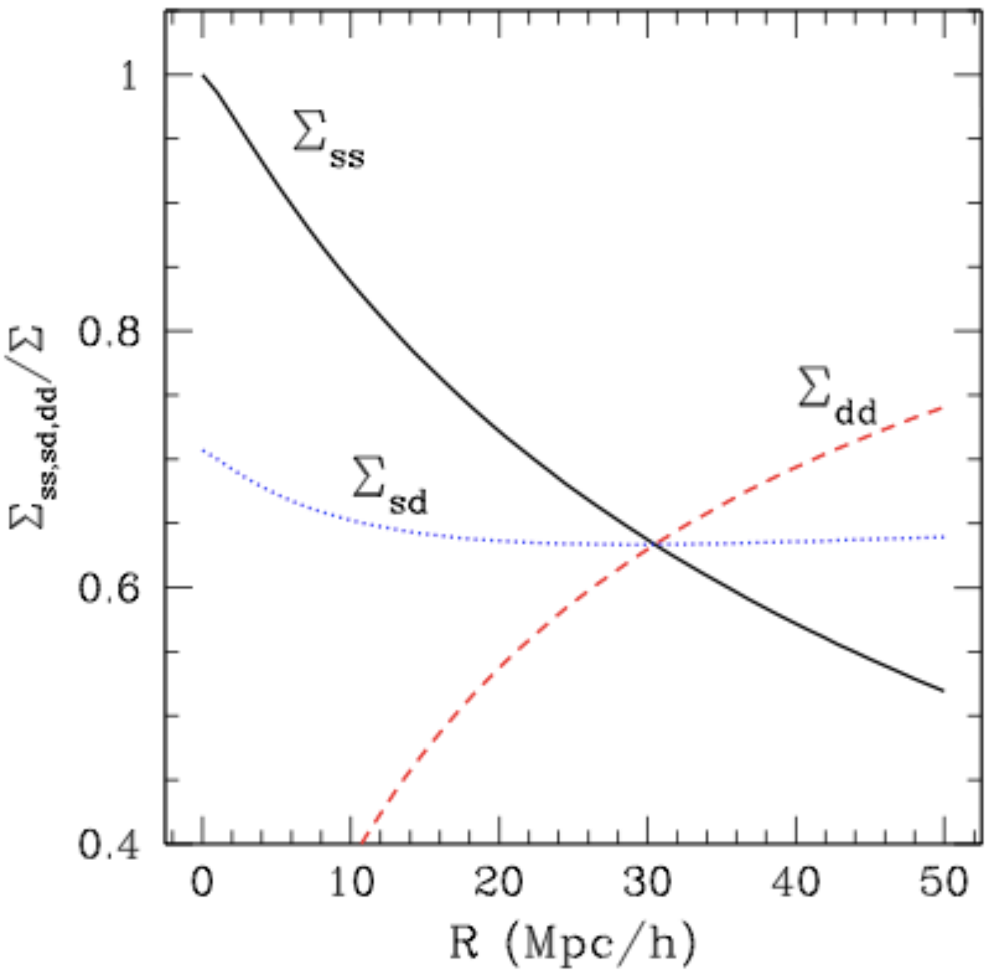
Schematically…

$$
\begin{aligned}
P_r(k) \; = & \Big\{ e^{-k^2\Sigma_{ss}^2/2}\mathcal{S}^2(k) \\
& + 2e^{-k^2\Sigma_{sd}^2/2}\mathcal{S}(k)\bar{\mathcal{S}}(k) \\
& + e^{-k^2\Sigma_{dd}^2/2}\bar{\mathcal{S}}^2(k)\Big\} P_L(k) + \ldots
\end{aligned}
$$

$$
\Sigma_{ss}^2 \equiv \frac{1}{3\pi^2}\int dq\, P_L(p)\mathcal{S}^2(p)\,,
$$

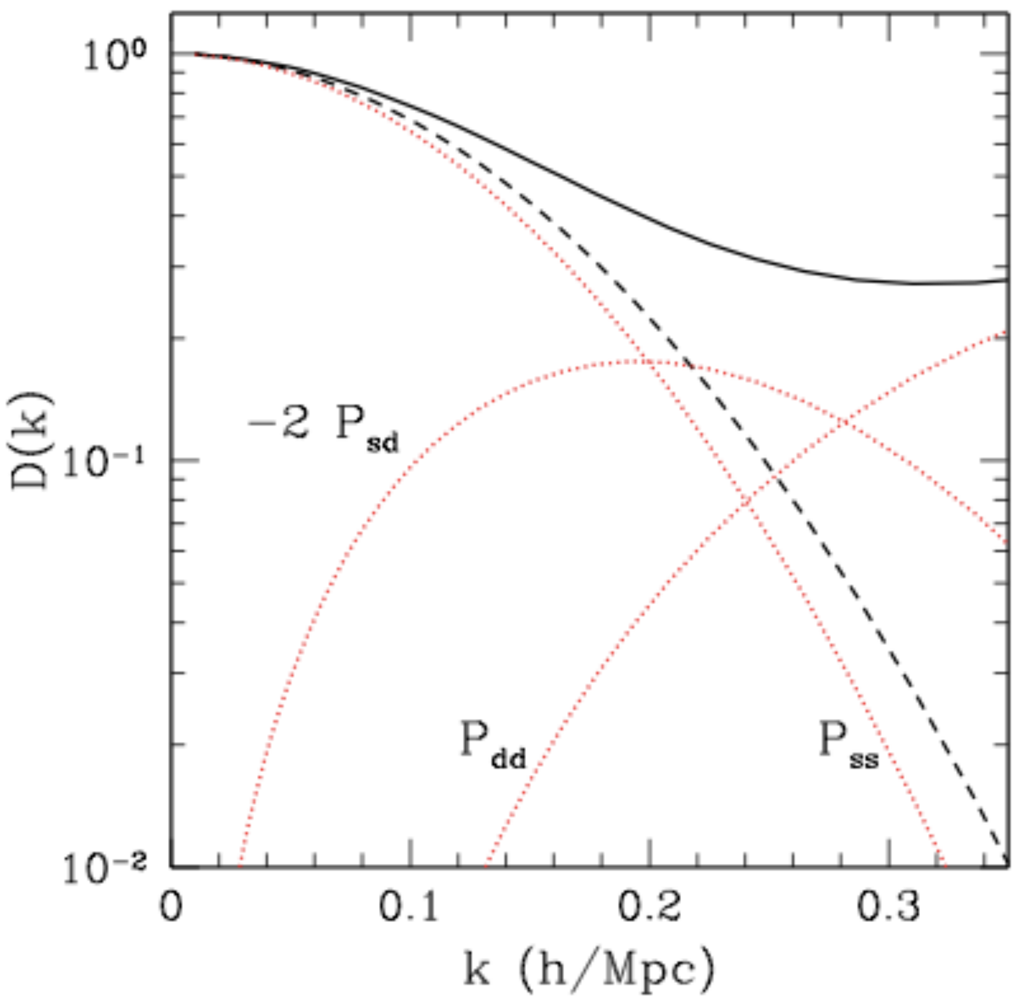
$$
\Sigma_{dd}^2 \equiv \frac{1}{3\pi^2}\int dq\, P_L(p)\bar{\mathcal{S}}^2(p)\,,
$$

Damping scales post reconstruction

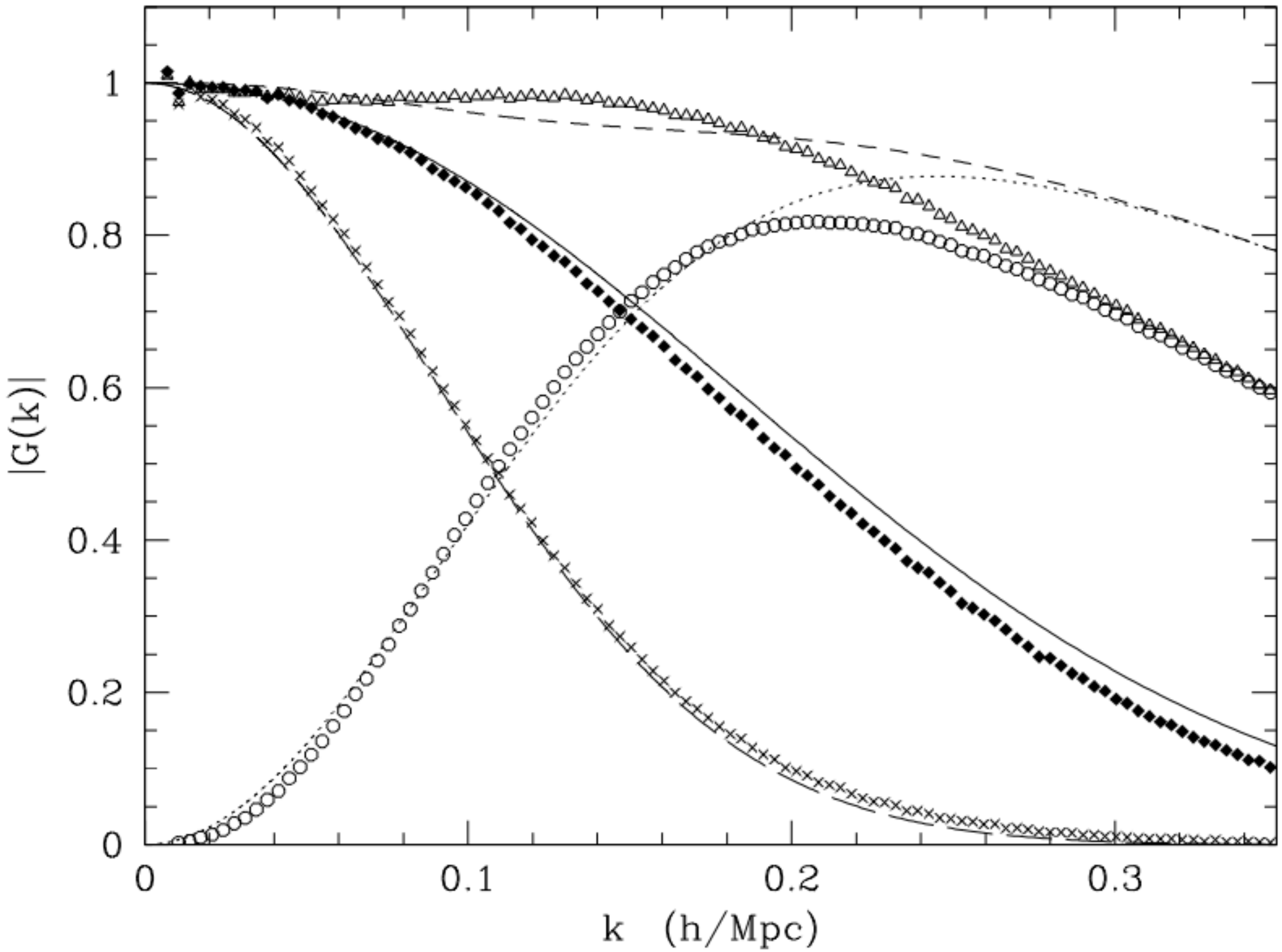


NP, White, Cohn 2009

Damping post reconstruction

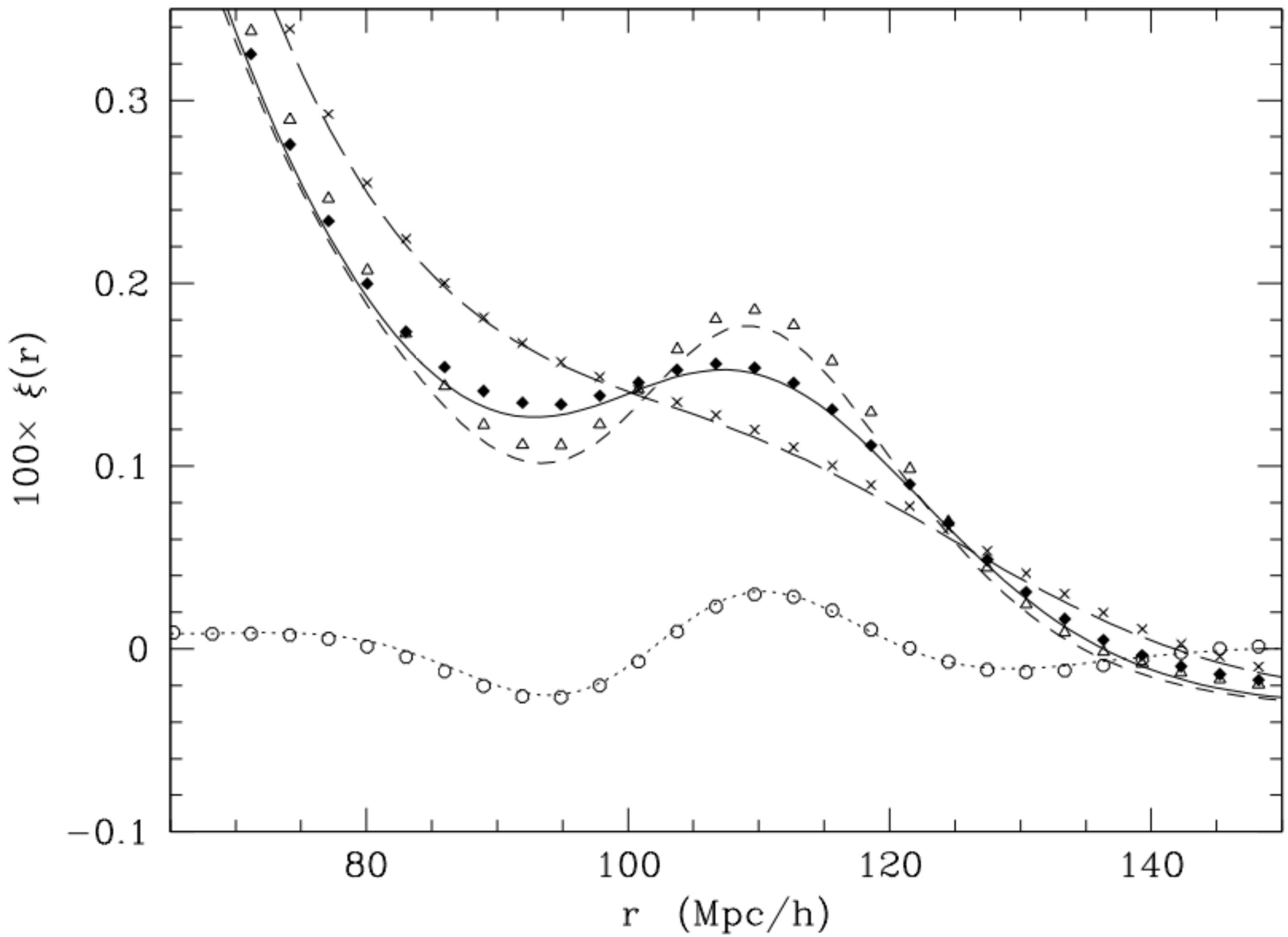


Damping of matter



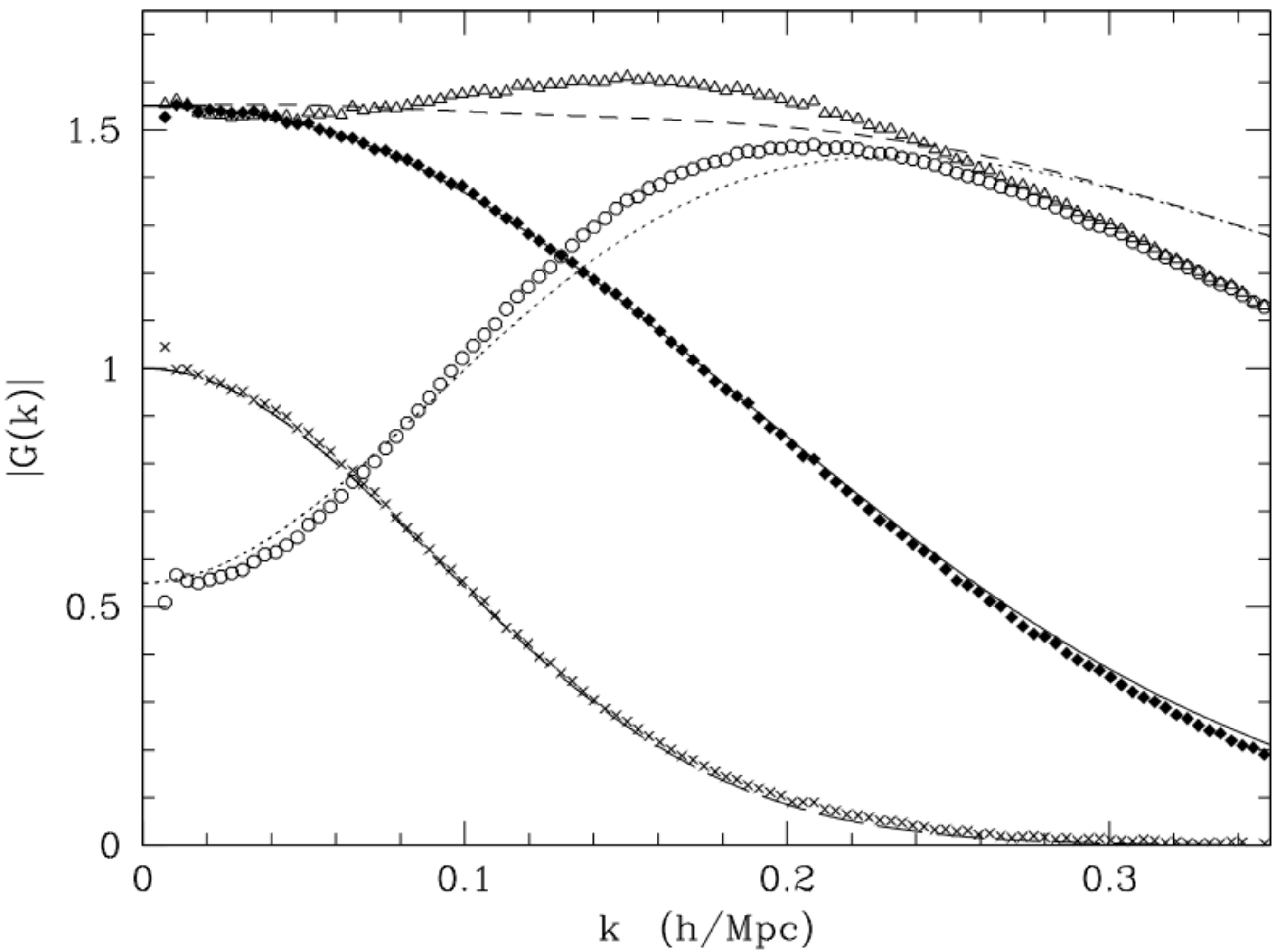
Noh, White, NP 2009

Reconstructed matter correlation fn



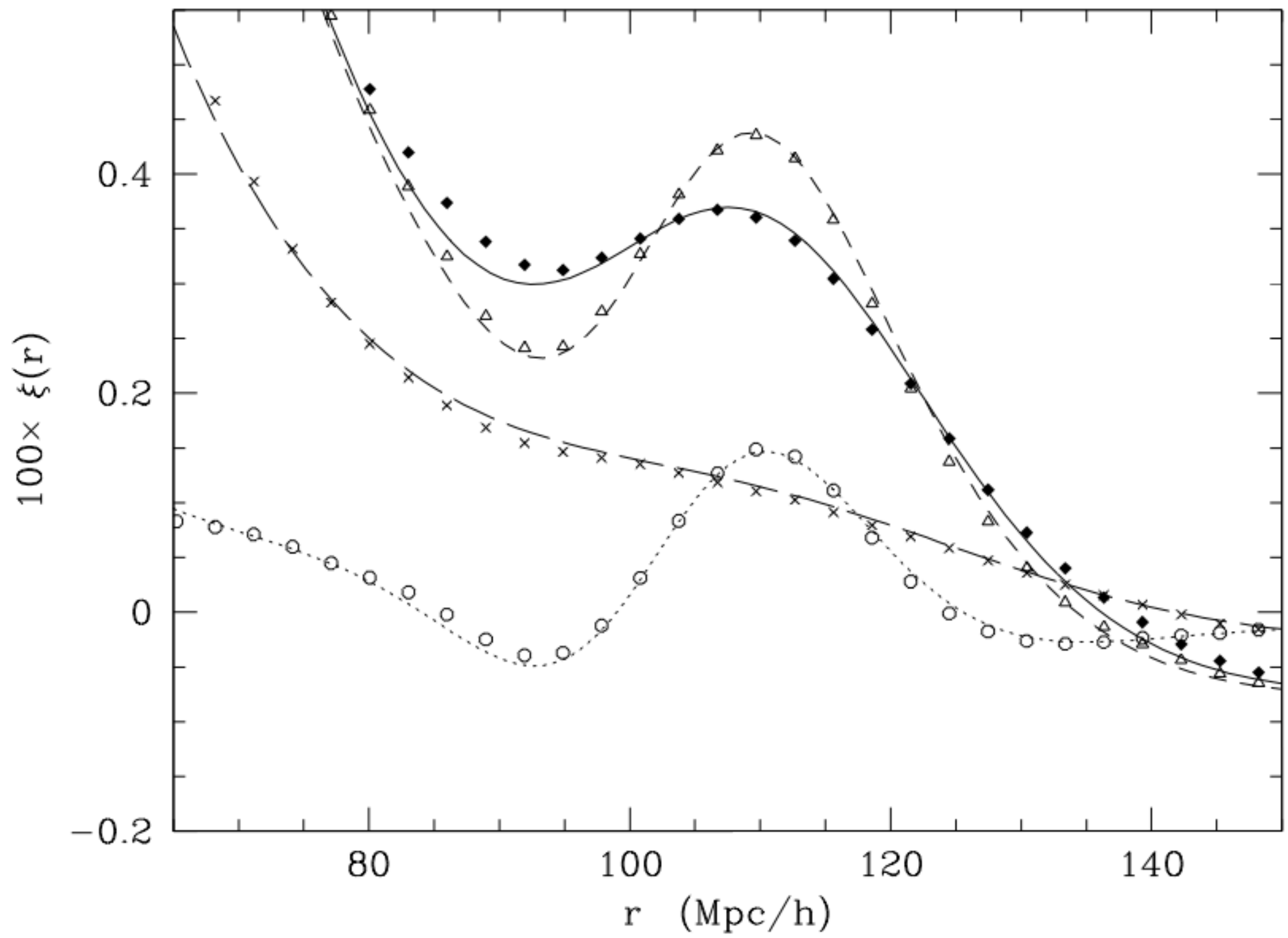
Noh, White, NP 2009

Damping of halos



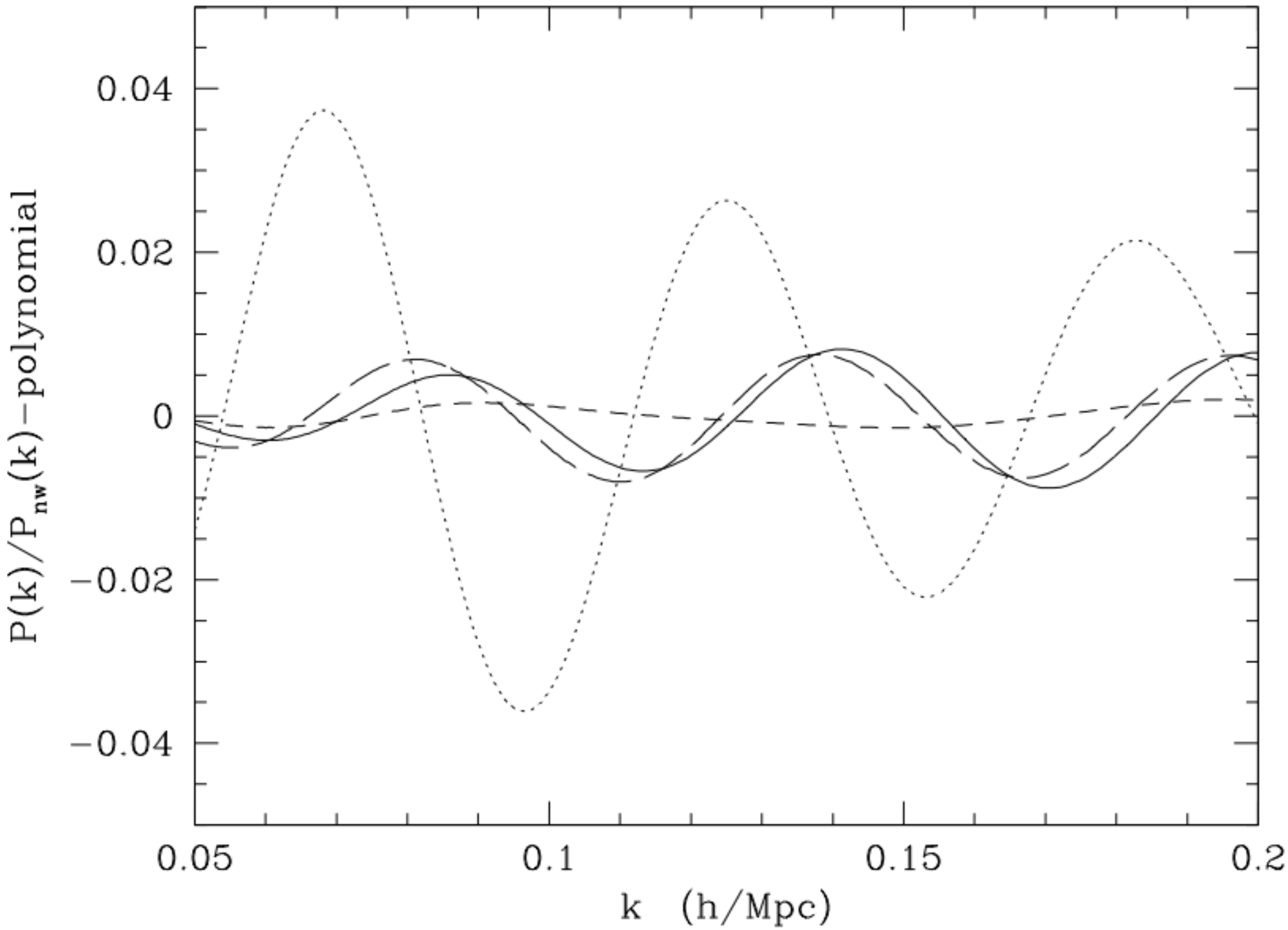
Noh, White, NP 2009

Reconstructed halo correlation fn



Noh, White, NP 2009

Reconstruction and shifts



Noh, White, NP 2009

A Physical Picture

- Blake and Zaldarriaga (2012) provide a physical picture for what is happening.
- Imagine splitting the density fluctuations in long and short wavelength terms
- For an long wavelength overdensity, imagine being an a somewhat closed Universe.
  - Scales are slightly contracted

$$\Gamma \equiv a_{\mathrm{curved}}(\delta_L)/a_0$$

$$1 + \delta_L = \left(\tfrac{1}{\Gamma}\right)^3$$

$$\xi_{S0}(r/\Gamma(\delta_L)) \approx \xi_{S0}((1+\tfrac{\delta_L}{3})r)$$

A Physical Picture : II

- For an long wavelength overdensity, imagine being an a somewhat closed Universe.
  - Growth is enhanced

$$(1+\tfrac{13}{21}\delta_L)^2(1+\delta_L)^2 \approx (1+\tfrac{68}{21}\delta_L)$$

  - Enhanced growth * correction to the true background

$$\begin{aligned}
\xi_S(r) &\approx (1+\frac{68}{21}\delta_L)\xi_{S0}((1+\frac{\delta_L}{3})r) \\
&\approx \xi_{S0}(r) + (\frac{68}{21}\xi_{S0}(r) + \frac{1}{3}r\xi'_{S0}(r))\delta_L \\
&+ \left[\frac{68\delta_L}{21}\frac{\delta_L}{3}r\xi'_{S0}(r)\right] + \ldots
\end{aligned}$$

A Physical Picture : III

- Why does reconstruction work?

$$F_2(\mathbf{k},\mathbf{q}) = \frac{17}{21} + \frac{1}{2}\frac{\mathbf{k}\cdot\mathbf{q}}{kq}\left(\frac{k}{q}+\frac{q}{k}\right) + \frac{2}{7}\left[\left(\frac{\mathbf{k}\cdot\mathbf{q}}{kq}\right)^2 - \frac{1}{3}\right].$$

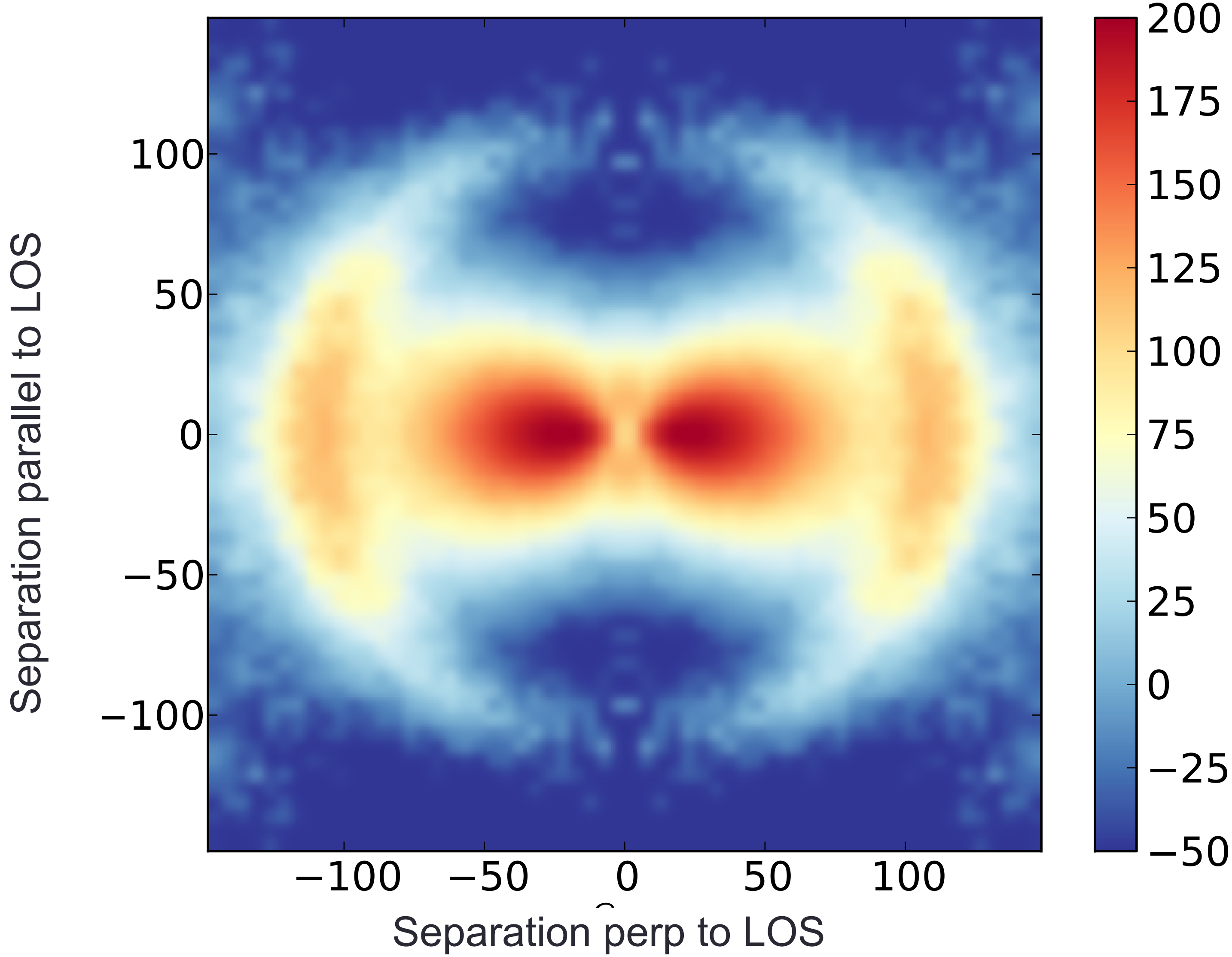
$$\delta(\mathbf{x}) = \delta_0(\mathbf{x}) + \underbrace{\mathbf{d}(\mathbf{x})\cdot\nabla\delta_0(\mathbf{x})}_{\text{shift}} + \underbrace{\frac{17}{21}\delta_0^2(\mathbf{x})}_{\text{growth}} + \underbrace{\frac{2}{7}K_{ij}(\mathbf{x})K_{ij}(\mathbf{x})}_{\text{anisotropy}} + \cdots$$

$$\mathbf{d}(\mathbf{x}) = -\int \frac{d^3q}{(2\pi)^3}\frac{i\mathbf{q}}{q^2}\delta_0(\mathbf{q})e^{i\mathbf{q}\cdot\mathbf{x}}$$

- The displacement field estimated from smoothed density field is approximately d(x)

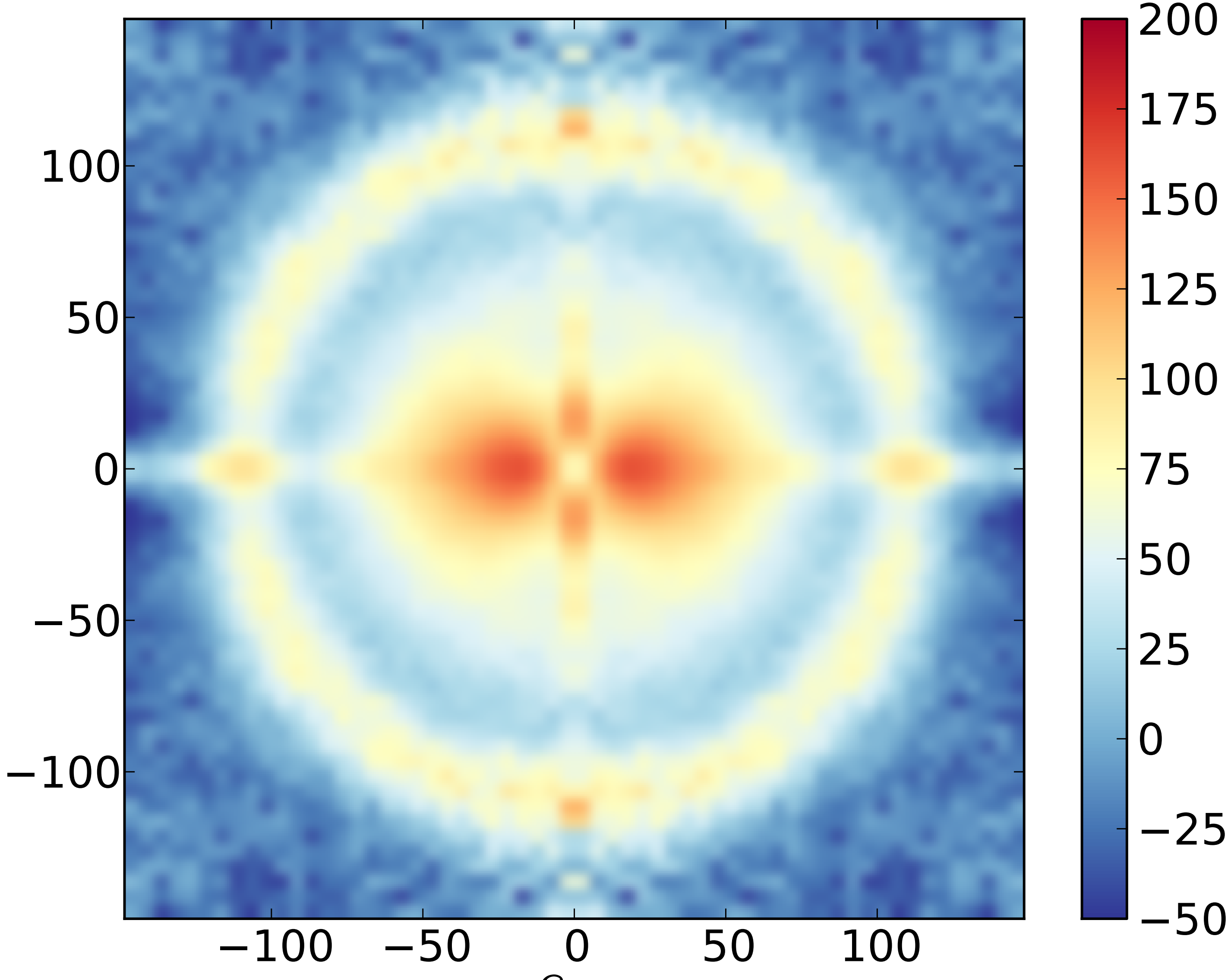
Simulations : Before

NP et al, 2012

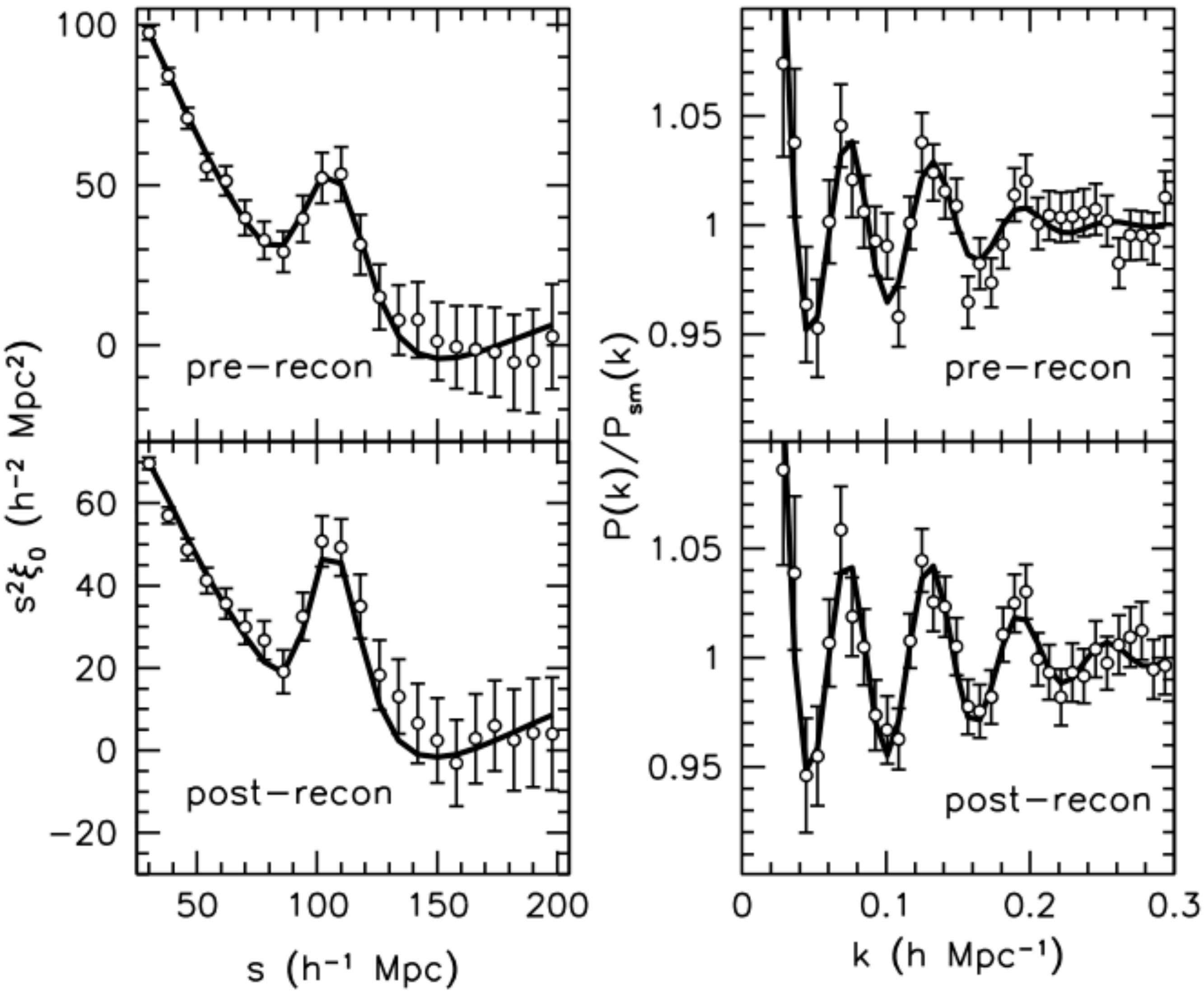


Simulations : After

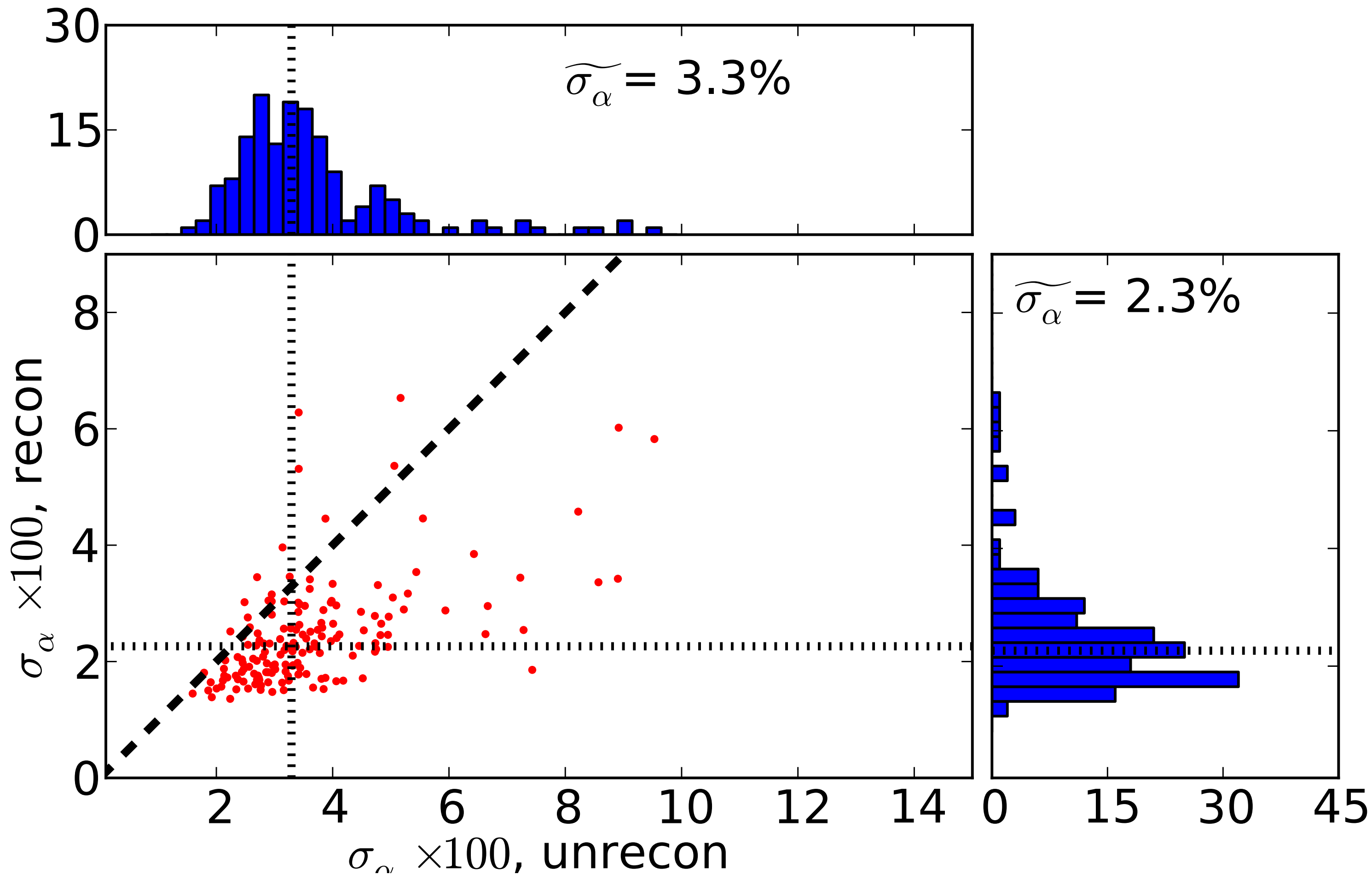
NP et al, 2012



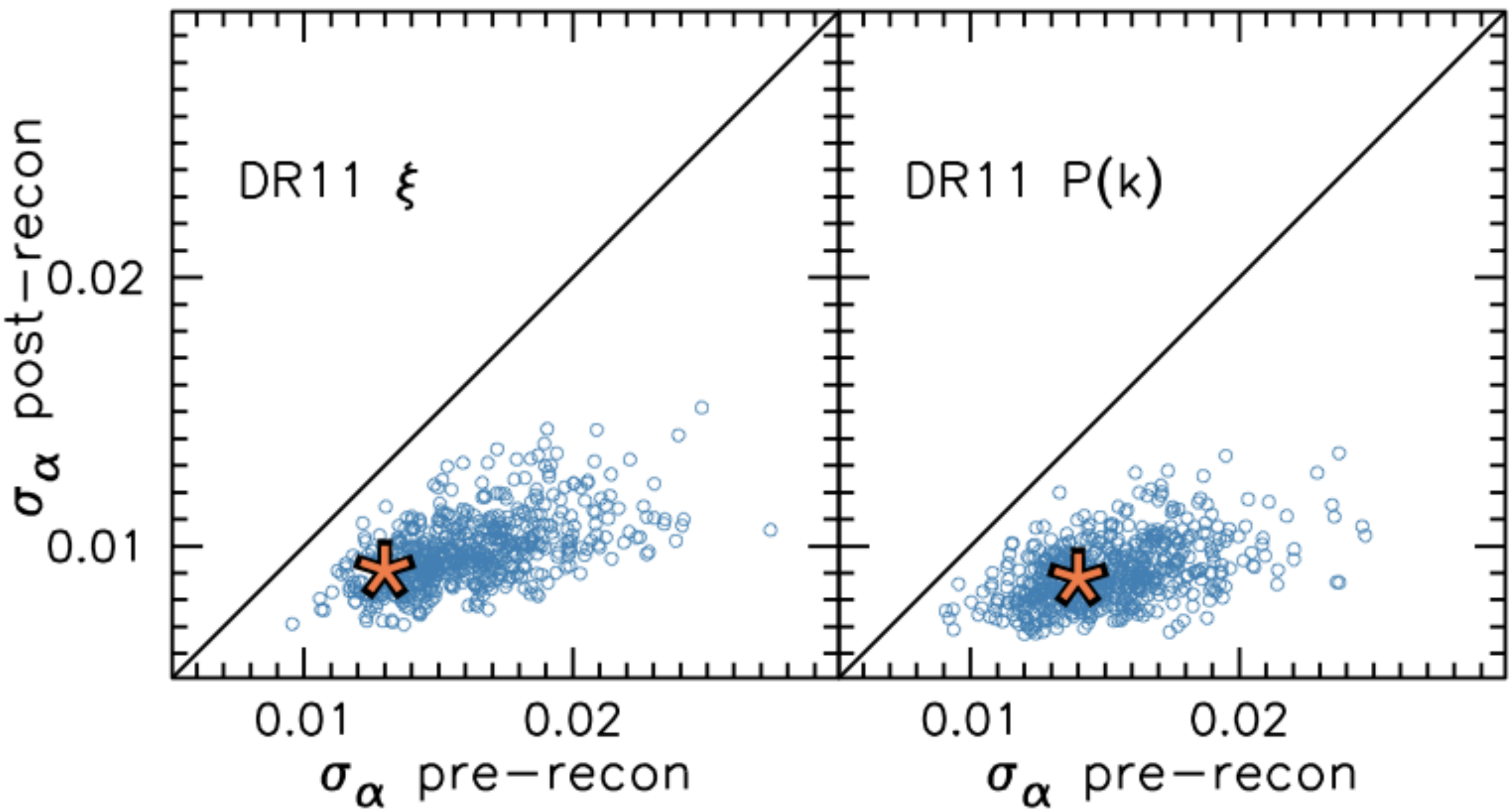
A Sharper Feature, More Oscillations



Errors in SDSS-II

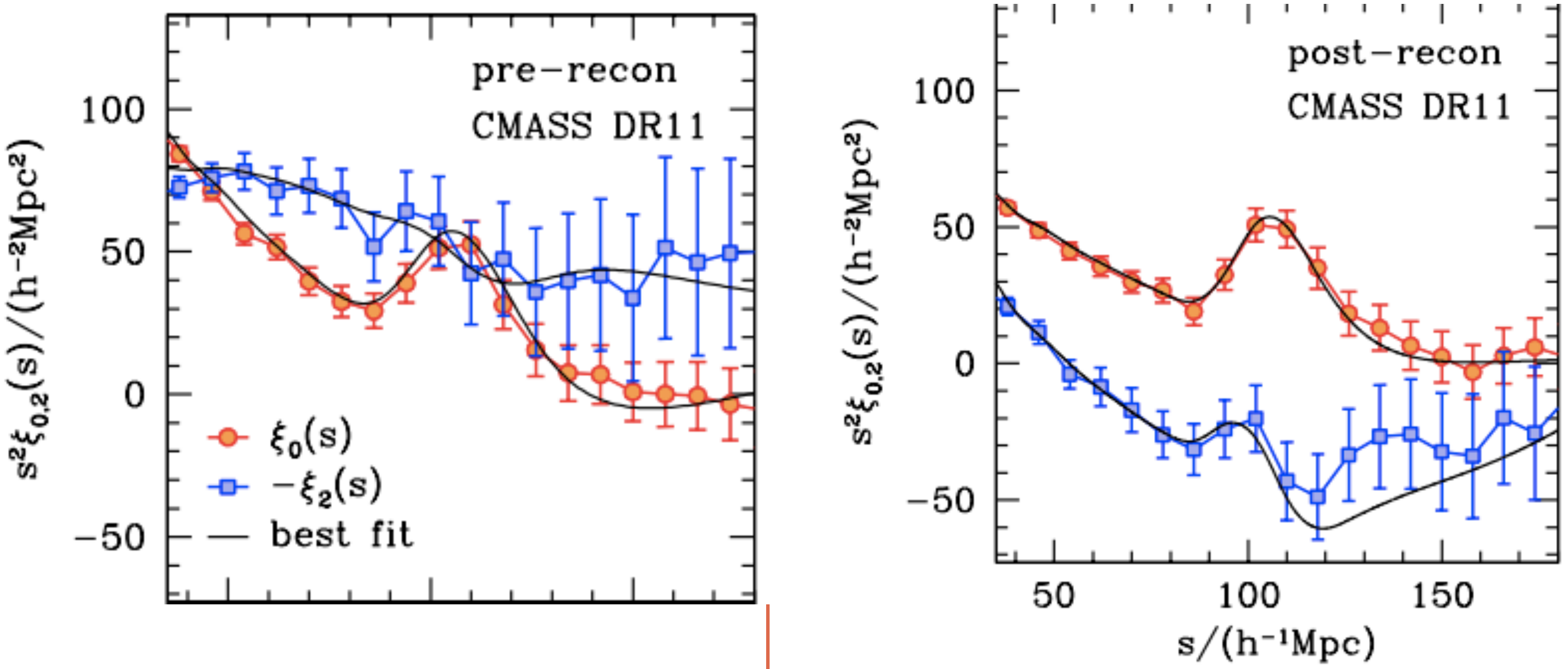


… and in DR11

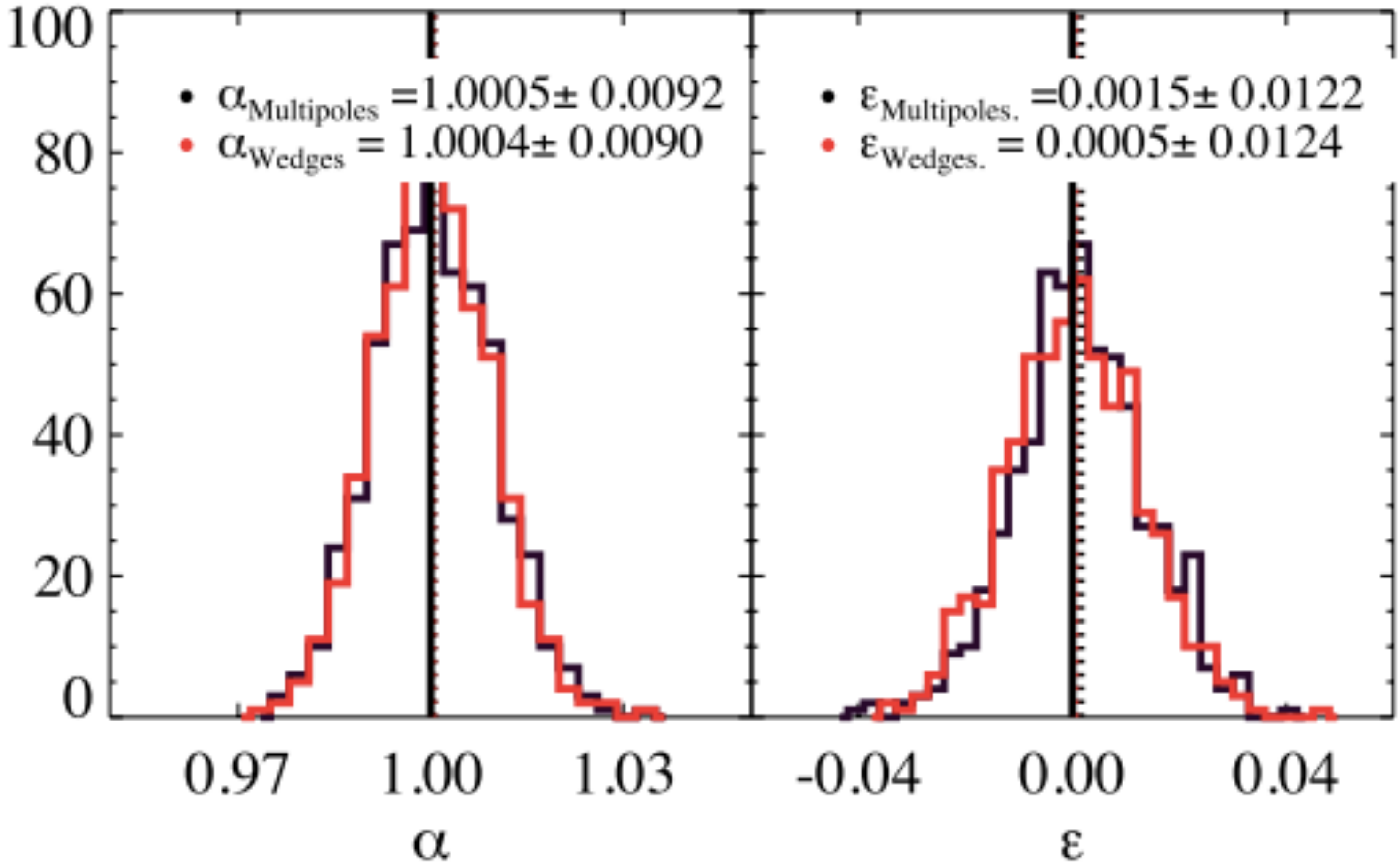


Anderson et al, 2013

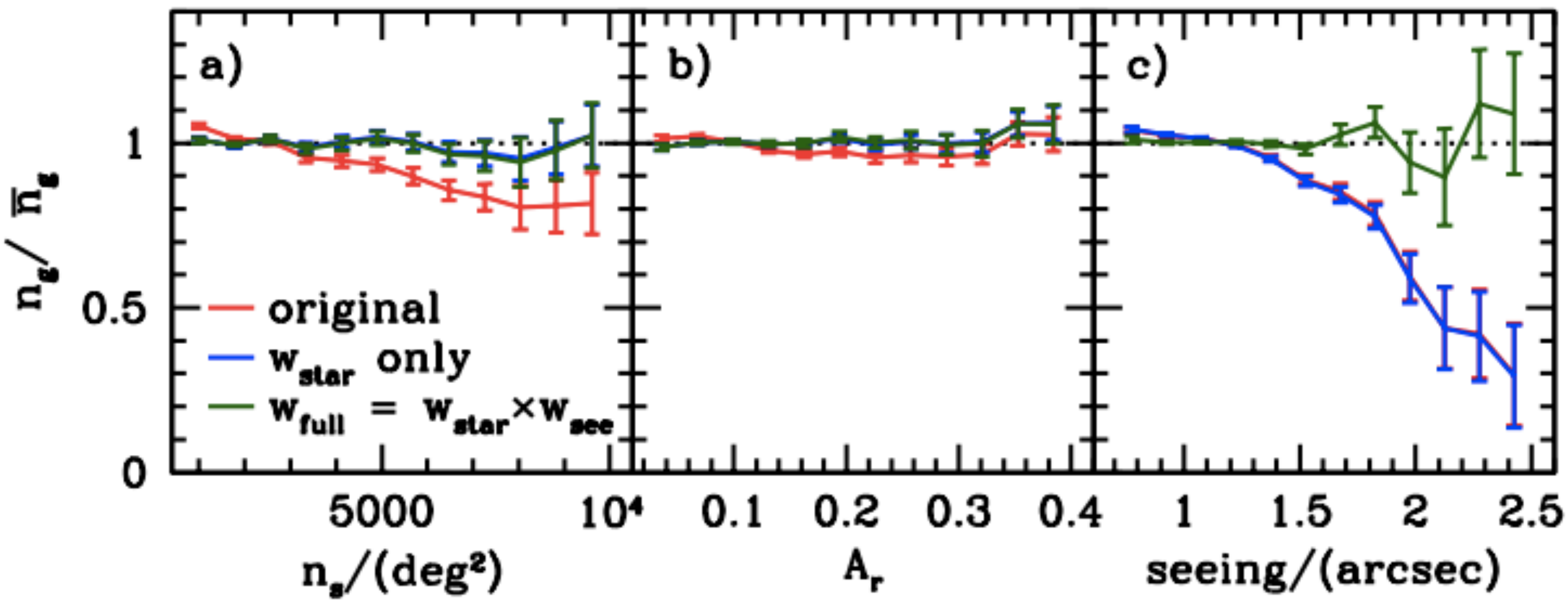
Angular dependence



Recovering distance



Observational systematics

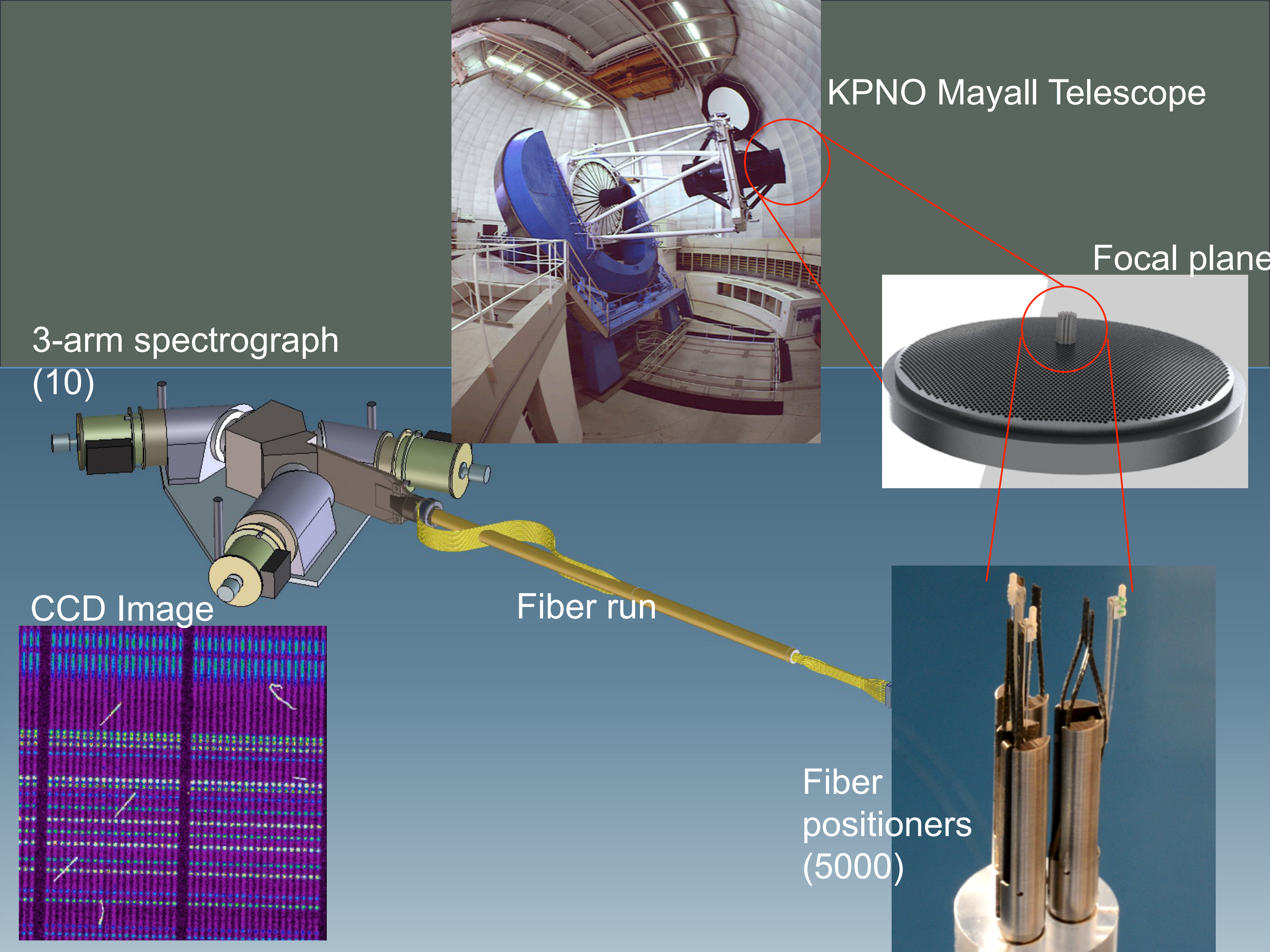


Ross et al 2014, Anderson et al 2014

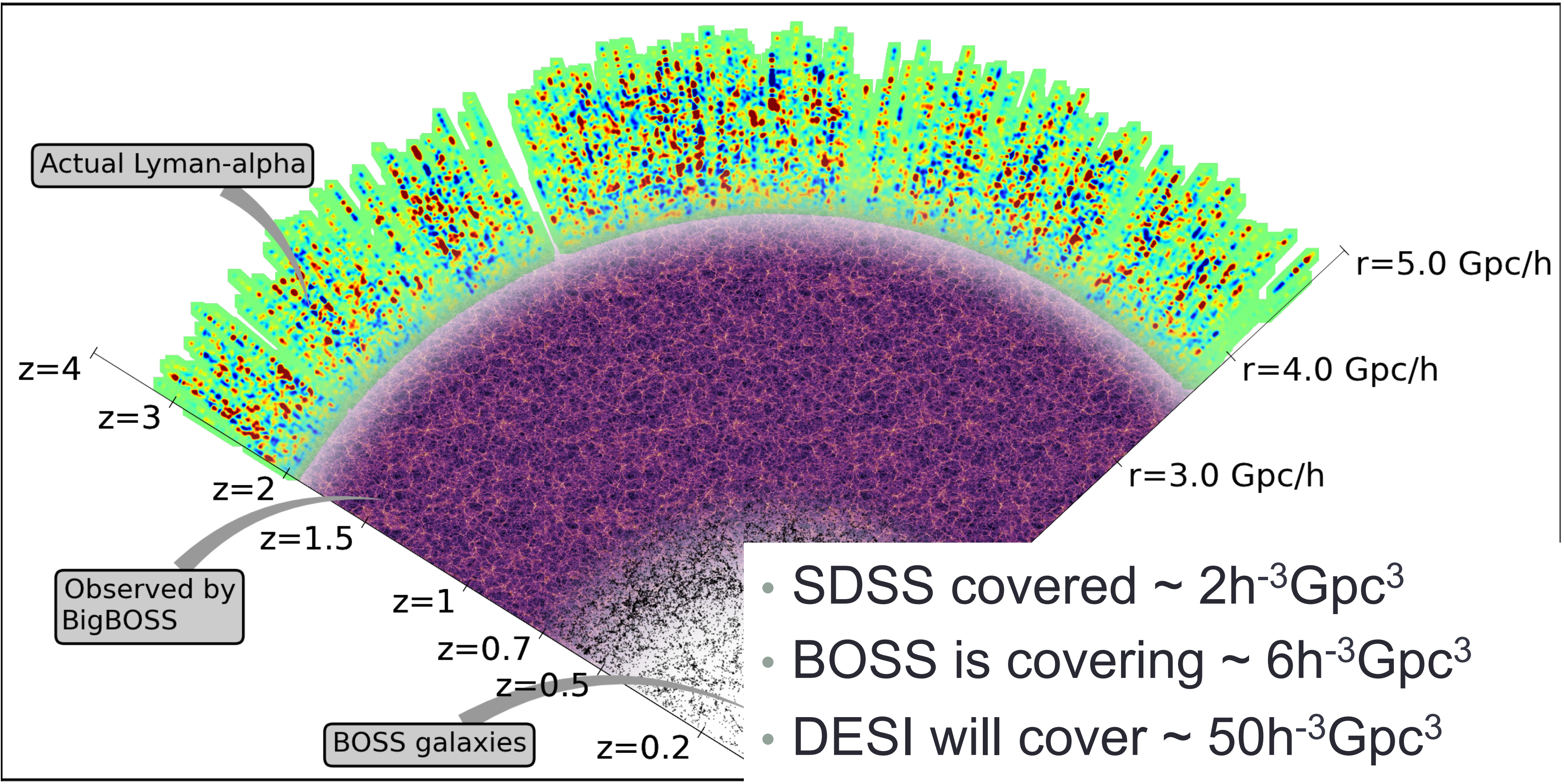
Designing next generation surveys

- Simplest picture : volume, number density, tracers

$$V_{\rm eff}(k,\mu) = \int \left[\frac{n(\mathbf{r})P(k,\mu)}{n(\mathbf{r})P(k,\mu)+1}\right]^2 d\mathbf{r}$$
$$= \left[\frac{nP(k,\mu)}{nP(k,\mu)+1}\right]^2 V_{\rm sur}$$
$$= \left[\frac{nP(k)(1+\beta\mu^2)^2}{nP(k)(1+\beta\mu^2)^2+1}\right]^2 V_{\rm sur},$$

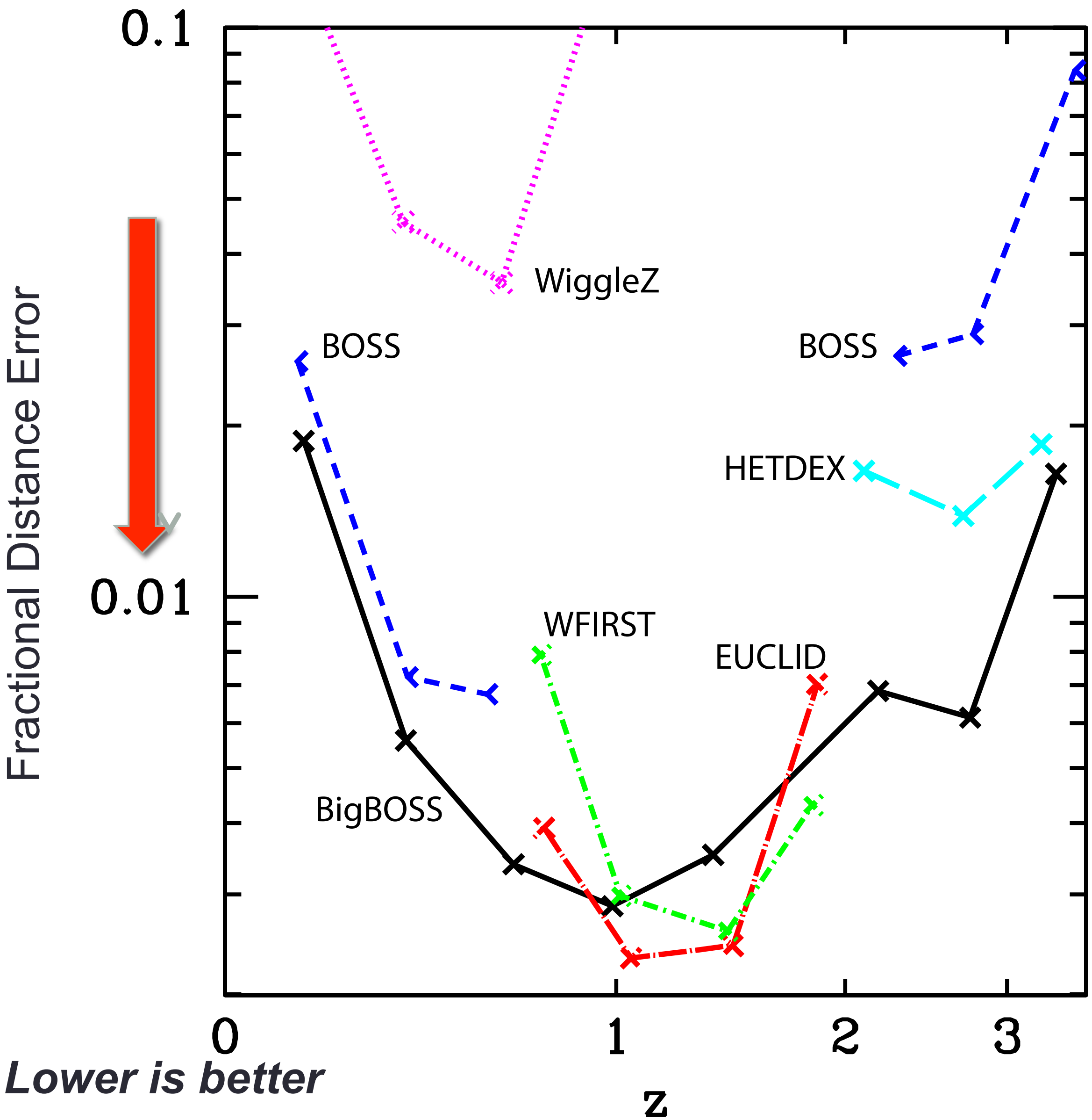

KPNO Mayall Telescope
Focal plane
3-arm spectrograph (10)
CCD Image
Fiber run
Fiber positioners (5000)

DESI is BIG



- SDSS covered ~ $2h^{-3}Gpc^3$
- BOSS is covering ~ $6h^{-3}Gpc^3$
- DESI will cover ~ $50h^{-3}Gpc^3$
- unprecedented volume

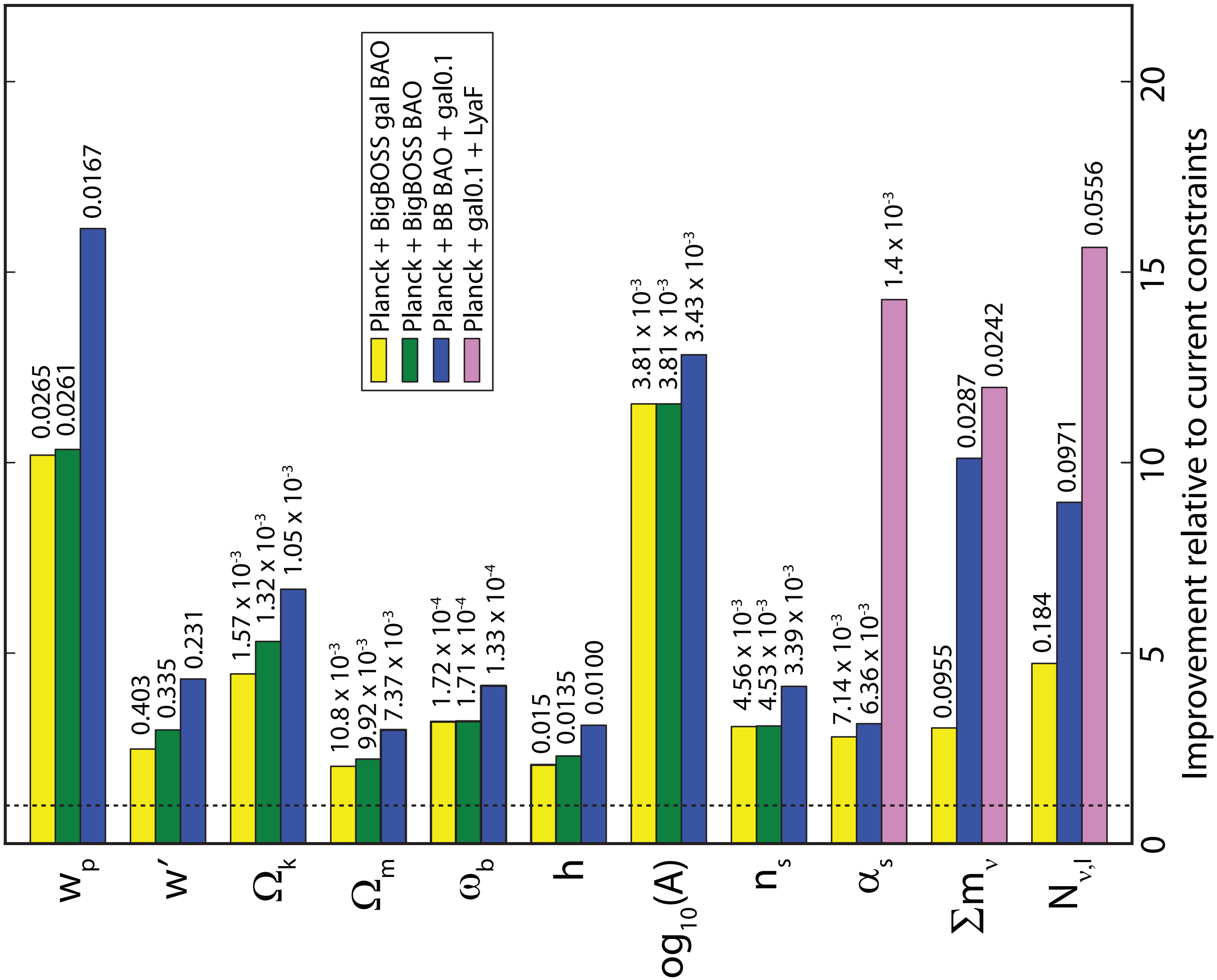
Distance constraints



- DESI has <1% distance errors over the widest redshift range
- Probe the expansion history over the widest redshift range

NASA WFIRST Science Definition Team

Beyond dark energy with DESI



Challenges

- Beyond BAO
- Systematics
  - Observational systematics
    - Survey design
    - Analysis techniques
  - Theoretical systematics
    - Nonlinearities
    - Galaxies
- Quantifying statistical precision
  - Errors rely on mock catalogs
- Estimators