Primordial nongaussianities I: cosmic microwave background

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Rio de Janeiro, August 2014
Outline

• Primordial nongaussianity
• Introduction and basic physics
• CMB temperature power spectrum and observables
• Primordial perturbations
• CMB Polarization: E and B modes
Gaussian random field, $\zeta(x)$

- normal distribution of values in real space, $\text{Prob}[\zeta(x)]$
  \[
  \text{Prob}(\zeta) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\zeta^2}{2\sigma^2}\right)
  \]

- defined entirely by power spectrum in Fourier space
  \[
  \left< \zeta_{\vec{k}} \zeta_{\vec{k}'}, \zeta_{\vec{k}''} \right> = (2\pi)^3 P_\zeta(k) \delta^3(\vec{k} + \vec{k}')
  \]

- bispectrum and (connected) higher-order correlations vanish
  \[
  \left< \zeta_{\vec{k}} \zeta_{\vec{k}'}, \zeta_{\vec{k}''} \right> = 0
  \]
non-Gaussian random field, $\zeta(x)$

anything else
Rocky Kolb

non-Rocky Kolb

[Images of Rocky Kolb and non-Rocky Kolb]
Primordial Gaussianity from inflation

- **Quantum fluctuations from inflation**
  - *ground state of simple harmonic oscillator*
  - *almost free field in almost de Sitter space*
  - *almost scale-invariant and almost Gaussian*

- **Power spectra probe background dynamics (H, ε, …)**
  \[
  \langle \zeta_{k_1} \zeta_{k_2} \rangle = (2\pi)^3 P_\zeta(k) \delta^3(k_1 + k_2) \ , \quad P_\zeta(k) \propto k^{n-4}
  \]
  - but, many different models, can produce similar power spectra

- **Higher-order correlations can distinguish different models**
  \[
  \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta^3(k_1 + k_2 + k_3)
  \]
  - non-Gaussianity ← non-linearity ← interactions = physics+gravity
# Many sources of non-Gaussianity

<table>
<thead>
<tr>
<th>Inflation Event</th>
<th>Source of Non-Gaussianity</th>
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<tbody>
<tr>
<td>Initial vacuum</td>
<td>Excited state</td>
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<tr>
<td>Sub-Hubble evolution</td>
<td>Higher-derivative interactions e.g. k-inflation, DBI, Galileons</td>
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<tr>
<td>Hubble-exit</td>
<td>Features in potential</td>
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<tr>
<td>Super-Hubble evolution</td>
<td>Self-interactions + gravity</td>
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<tr>
<td>End of inflation</td>
<td>Tachyonic instability</td>
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<tr>
<td>(p)Reheating</td>
<td>Modulated (p)reheating</td>
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<tr>
<td>After inflation</td>
<td>Curvaton decay</td>
</tr>
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<td></td>
<td>Magnetic fields</td>
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</table>

**Primordial non-Gaussianity**

<table>
<thead>
<tr>
<th>Anisotropy Type</th>
<th>Phenomenon</th>
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<tbody>
<tr>
<td>Primary anisotropies</td>
<td>Last-scattering</td>
</tr>
<tr>
<td>Secondary anisotropies</td>
<td>ISW/lensing + foregrounds</td>
</tr>
</tbody>
</table>
Many shapes for primordial bispectra

- **local type** (Komatsu & Spergel 2001)
  - local in real space
  - max for squeezed triangles: $k < k', k''$
  \[
  B_\zeta(k_1, k_2, k_3) \propto \left( \frac{1}{k_1^3 k_2^3} + \frac{1}{k_2^3 k_3^3} + \frac{1}{k_3^3 k_1^3} \right)
  \]

- **equilateral type** (Creminelli et al 2005)
  - peaks for $k_1 \sim k_2 \sim k_3$
  \[
  B_\zeta(k_1, k_2, k_3) \propto \left( \frac{3(k_1 + k_2 - k_3)(k_2 + k_3 - k_1)(k_3 + k_1 - k_2)}{k_1^3 k_2^3 k_3^3} \right)
  \]

- **orthogonal type** (Senatore et al 2009)
  - independent of local + equilateral shapes
  \[
  B_\zeta(k_1, k_2, k_3) \propto \left( \frac{81}{k_1 k_2 k_3(k_1 + k_2 + k_3)^3} \right)
  \]

- **separable basis** (Ferguson et al 2008)
Evolution of the universe

Black body spectrum observed by COBE

- close to thermal equilibrium:
  temperature today of 2.726K (~ 3000K at z ~ 1000 because $\nu \sim (1+z)$)

Residuals Mather et al 1994
Observations: the microwave sky today

(almost) uniform 2.726K blackbody

Dipole (local motion)

O(10^{-5}) perturbations (+galaxy)

Source: NASA/WMAP Science Team
Can we predict the primordial perturbations?

- Maybe..

Quantum Mechanics
“waves in a box” calculation
vacuum state, etc…

Inflation
make $>10^{30}$ times bigger

After inflation
Huge size, amplitude $\sim 10^{-5}$
Perturbation evolution – what we actually observe
CMB monopole source till 380 000 yrs (last scattering), linear in conformal time
scale invariant primordial adiabatic scalar spectrum

photon/baryon plasma + dark matter, neutrinos

Characteristic scales: sound wave travel distance; diffusion damping length
Observed $\Delta T$ as function of angle on the sky

Diagram showing the relationship between the last scattering surface, the observer, and the events of recombination and $z=1000$ with $\lambda \sim k^{-1}$ and $\theta \sim t^{-1}$. 
Calculation of theoretical perturbation evolution

Perturbations $O(10^{-5})$

Simple linearized equations are very accurate (except small scales)

Can use real or Fourier space

Fourier modes evolve independently: simple to calculate accurately

Physics Ingredients

• Thomson scattering (non-relativistic electron-photon scattering)
  - tightly coupled before recombination: ‘tight-coupling’ approximation
    (baryons follow electrons because of very strong e-m coupling)
• Background recombination physics (Saha/full multi-level calculation)
• Linearized General Relativity
• Boltzmann equation (how angular distribution function evolves with scattering)
CMB power spectrum $C_l$

- Theory: Linear physics + Gaussian primordial fluctuations

$$a_{lm} = \int d\Omega \Delta T Y_{lm}^*$$

Theory prediction $C_l = \langle |a_{lm}|^2 \rangle$

- variance (average over all possible sky realizations)
- statistical isotropy implies independent of $m$

Initial conditions + cosmological parameters

linearized GR + Boltzmann equations

CMBFAST: cmbfast.org
CAMB: camb.info
CMBEASY: cmbeasy.org
COSMICS, etc.
Sources of CMB anisotropy

**Sachs Wolfe:**
- Potential wells at last scattering cause redshifting as photons climb out

**Photon density perturbations:**
- Over-densities of photons look hotter

**Doppler:**
- Velocity of photon/baryons at last scattering gives Doppler shift

**Integrated Sachs Wolfe:**
- Evolution of potential along photon line of sight:
  net red- or blue-shift as photon climbs in and out of varying potential wells

**Others:**
- Photon quadupole/polarization at last scattering, second-order effects, etc.
CMB temperature power spectrum
Primordial perturbations + later physics

Acoustic oscillations
Diffusion damping

Why $C_l$ oscillations?

Think in k-space: modes of different size

- Co-moving Poisson equation: $(k/a)^2 \Phi = \kappa \delta \rho / 2$
  - potentials approx constant on super-horizon scales
  - radiation domination $\rho \sim 1/a^4$

  $\Rightarrow \delta \rho / \rho \sim k^2 a^2 \Phi$
  $\Rightarrow$ since $\Phi \sim$ constant, super-horizon density perturbations grow $\sim a^2$

- After entering horizon pressure important: perturbation growth slows, then bounces back

  $\Rightarrow$ series of acoustic oscillations (sound speed $\sim c/\sqrt{3}$)

- CMB anisotropy (mostly) from a surface at fixed redshift: phase of oscillation at time of last scattering depends on time since entering the horizon

  $\Rightarrow$ k-dependent oscillation amplitude in the observed CMB
Fig. 3. Evolution of the combination $\delta_\gamma/4 + \psi$ (top left) and the photon velocity $v_\gamma$ (bottom left) which determine the temperature anisotropies produced at last scattering (denoted by the arrow at $\eta_*$). Three modes are shown with wavenumbers $k = 0.001, 0.1$ and $0.2$ $\text{Mpc}^{-1}$, and the initial conditions are adiabatic. The fluctuations at the time of last scattering are shown as a function of linear scale in the right-hand plot.
Contributions to temperature $C_l$

Challinor: astro-ph/0403344
What can we learn from the CMB?

• **Initial conditions**
  What types of perturbations, power spectra, distribution function (Gaussian?);
  => learn about inflation or alternatives.
  (distribution of $\Delta T$; power as function of scale; polarization and correlation)

• **What and how much stuff**
  Matter densities ($\Omega_b$, $\Omega_{cdm}$); neutrino mass
  (details of peak shapes, amount of small scale damping)

• **Geometry and topology**
  global curvature $\Omega_K$ of universe; topology
  (angular size of perturbations; repeated patterns in the sky)

• **Evolution**
  Expansion rate as function of time; reionization
  - Hubble constant $H_0$; dark energy evolution $w = \text{pressure/density}$
  (angular size of perturbations; $l < 50$ large scale power; polarization)

• **Astrophysics**
  S-Z effect (clusters), foregrounds, etc.
• Cosmic Variance: only one sky

Use estimator for variance:

\[ C_{l}^{\text{obs}} = \frac{1}{2l + 1} \sum_{m} |a_{lm}|^2 \]

Assume \( a_{lm} \) gaussian:

\( C_{l}^{\text{obs}} \sim \chi^2 \) with \( 2l + 1 \) d.o.f.

“Cosmic Variance”

\[ \left\langle |\Delta C_{l}^{\text{obs}}|^2 \right\rangle \approx \frac{2C_{l}^2}{2l + 1} \]

\( P(C_{l} \mid C_{l}^{\text{obs}}) \)

- inverse gamma distribution
  (+ noise, sky cut, etc).

Cosmic variance gives fundamental limit on how much we can learn from CMB
How to generate CMB polarization?
How to generate CMB polarization?

(Isotropy)

(Thomson Scattering)

(No Polarization)

(Wayne Hu, CMB Tutorials)
How to generate CMB polarization?

- Isotropy
- Thomson Scattering
- No Polarization

- Quadrupole Anisotropy
- Thomson Scattering
- Linear Polarization

(Wayne Hu, CMB Tutorials)
Polarization: Stokes’ Parameters

Q → -Q, U → -U under 90 degree rotation
Q → U, U → -Q under 45 degree rotation

Spin-2 field $Q + i U$
or Rank 2 trace free symmetric tensor

$$P = \begin{pmatrix} Q & U \\ U & -Q \end{pmatrix}$$

$$\theta = \frac{1}{2} \tan^{-1} \frac{U}{Q}$$

$$\sqrt{Q^2 + U^2}$$
E and B mode patterns

E-mode

Q > 0 \hspace{0.2cm} U = 0

E < 0

Q < 0 \hspace{0.2cm} U = 0

E > 0

Unchanged if seen through the mirror

B-mode

Q = 0 \hspace{0.2cm} U < 0

B < 0

Q = 0 \hspace{0.2cm} U > 0

B > 0

Pattern reverses if seen through the mirror

Seljak and Zaldarriaga, 1998
E and B harmonics

- Expand scalar $P_E$ and $P_B$ in spherical harmonics
- Expand $P_{ab}$ in tensor spherical harmonics

$$P_{ab} = \frac{1}{\sqrt{2}} \sum_{lm} \left( E_{lm} Y_{lm}^G Y_{ab} + B_{lm} Y_{lm}^C Y_{ab} \right)$$

$$E_{lm} = \sqrt{2} \int_{4\pi} dS Y_{lm}^G Y_{ab}^* \mathcal{P}_{ab} \quad \quad B_{lm} = \sqrt{2} \int_{4\pi} dS Y_{lm}^C Y_{ab}^* \mathcal{P}_{ab}$$

Harmonics are orthogonal over the full sky:

E/B decomposition is exact and lossless on the full sky

Zaldarriaga, Seljak: astro-ph/9609170
Kamionkowski, Kosowsky, Stebbins: astro-ph/9611125
CMB Polarization Signals

- E polarization from scalar, vector and tensor modes

- B polarization only from vector and tensor modes (curl grad = 0) + non-linear scalars

Average over possible realizations (statistically isotropic):

\[
\langle E_{l'm'}^* E_{lm} \rangle = \delta_{ll'} \delta_{m'm} C_l^{EE} \quad \langle B_{l'm'}^* B_{lm} \rangle = \delta_{ll'} \delta_{m'm} C_l^{BB}
\]

Parity symmetric ensemble:

\[
\langle E_{l'm'}^* B_{lm} \rangle = 0
\]

Power spectra contain all the useful information if the field is Gaussian
Scalar adiabatic mode

E polarization only
correlation to temperature T-E
Primordial Gravitational Waves (tensor modes)

- **Well motivated by some inflationary models**
  - Amplitude measures inflaton potential at horizon crossing
  - distinguish models of inflation

- **Observation would rule out other models**
  - ekpyrotic scenario predicts exponentially small amplitude
  - small also in many models of inflation, esp. two field e.g. curvaton

- **Weakly constrained from CMB temperature anisotropy**
  - cosmic variance limited to 10%
  - degenerate with other parameters (tilt, reionization, etc)

Look at CMB polarization: ‘B-mode’ smoking gun

Has BICEP2 observed it?
CMB polarization from primordial gravitational waves (tensors)

- Amplitude of tensors unknown
- Clear signal from B modes – there are none from scalar modes
- Tensor B is always small compared to adiabatic E

Seljak, Zaldarriaga: astro-ph/9609169
Reionization

Ionization since \( z \approx 6-20 \) scatters CMB photons

Temperature signal similar to tensors

Quadrupole at reionization implies large scale polarization signal

Measure optical depth with WMAP T-E correlation
Other non-linear effects

- **Thermal Sunyaev-Zeldovich**
  Inverse Compton scattering from hot gas: frequency dependent signal

- **Kinetic Sunyaev-Zeldovich (kSZ)**
  Doppler from bulk motion of clusters; patchy reionization; (almost) frequency independent signal

- **Ostriker-Vishniac (OV)**
  same as kSZ but for early linear bulk motion

- **Rees-Sciama**
  Integrated Sachs-Wolfe from evolving non-linear potentials: frequency independent

- **Gravitational lensing**
  discussed on friday
Newtonian potential a Gaussian random field
\[ \Phi(x) = \phi_G(x) \]
Newtonian potential a local function of Gaussian random field
\[ \Phi(x) = \phi_G(x) + f_{NL} \left( \phi_G^2(x) - \langle \phi_G^2 \rangle \right) \]

\[ f_{NL} = +3000 \]

\[ \Delta T/T \approx -\Phi/3 \], so positive \( f_{NL} \) \( \Rightarrow \) more cold spots in CMB
Newtonian potential a local function of Gaussian random field
\[ \Phi(x) = \phi_G(x) + f_{NL} ( \phi_G^2(x) - \langle \phi_G^2 \rangle ) \]

\( f_{NL} = -3000 \)

\[ \Delta T/T \approx -\Phi/3 \], so negative \( f_{NL} \) \( \Rightarrow \) more hot spots in CMB

Liguori, Matarrese and Moscardini (2003)
Newtonian potential a local function of **Gaussian random field**

\[ \Phi(x) = \phi_G(x) + f_{NL} (\phi_G^2(x) - \langle \phi_G^2 \rangle) \]

⇒ **Large-scale modulation of small-scale power**

*split Gaussian field into long (L) and short (s) wavelengths*

\[ \phi_G(X+x) = \phi_L(X) + \phi_s(x) \]

*two-point function on small scales for given \( \phi_L \)

\[ \langle \Phi(x_1) \Phi(x_2) \rangle_L = (1+4f_{NL} \phi_L) \langle \phi_s(x_1) \phi_s(x_2) \rangle + \ldots \]

\[ i.e., \text{ inhomogeneous modulation of small-scale power} \]

\[ P(k, X) \rightarrow [1 + 4f_{NL} \phi_L(X)] P_s(k) \]

but \( f_{NL} < 5 \) so any effect must be \( 10^{-4} \)
Optimal estimation from CMB data

\[ \hat{B}_{\ell_1 \ell_2 \ell_3} = \int d^2 \hat{n} T_{\ell_1}(\hat{n}) T_{\ell_2}(\hat{n}) T_{\ell_3}(\hat{n}) , \]

where the filtered maps \( T_\ell(\hat{n}) \) are defined as:

\[ T_\ell(\hat{n}) = \sum_m a_{\ell m} Y_{\ell m}(\hat{n}) . \]

\[ \hat{f}_{NL} = \frac{6}{N} \sum_{\ell_1 \leq \ell_2 \leq \ell_3} \frac{B_{\ell_1 \ell_2 \ell_3}^{th} (B_{\ell_1 \ell_2 \ell_3}^{obs} - B_{\ell_1 \ell_2 \ell_3}^{lin})}{V_{\ell_1 \ell_2 \ell_3}} . \]

\[ V_{\ell_1 \ell_2 \ell_3} = g_{\ell_1 \ell_2 \ell_3} h_{\ell_1 \ell_2 \ell_3}^2 C_{\ell_1} C_{\ell_2} C_{\ell_3} . \]

\[ h_{\ell_1 \ell_2 \ell_3} = \sqrt{\frac{(2\ell_1 + 1)(2\ell_2 + 1)(2\ell_3 + 1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} . \]
Planck Paper XXIV constraints: no evidence for primordial nongaussianity

<table>
<thead>
<tr>
<th>Shape</th>
<th>Independent KSW</th>
<th>Independent Binned</th>
<th>Independent Modal</th>
<th>ISW-lensing subtracted KSW</th>
<th>ISW-lensing subtracted Binned</th>
<th>ISW-lensing subtracted Modal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local</td>
<td>9.8 ± 5.8</td>
<td>9.2 ± 5.9</td>
<td>8.3 ± 5.9</td>
<td>2.7 ± 5.8</td>
<td>2.2 ± 5.9</td>
<td>1.6 ± 6.0</td>
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<tr>
<td>Equilateral</td>
<td>-37 ± 75</td>
<td>-20 ± 73</td>
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<td>-25 ± 73</td>
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<td>Orthogonal</td>
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<td>-17 ± 41</td>
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<td>NILC</td>
<td>11.6 ± 5.8</td>
<td>10.5 ± 5.8</td>
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<tr>
<td>Local</td>
<td>-41 ± 76</td>
<td>-31 ± 73</td>
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<td>-38 ± 73</td>
<td>-20 ± 78</td>
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<td>-74 ± 40</td>
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<td>-53 ± 40</td>
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<td>-37 ± 43</td>
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<td>SEVEM</td>
<td>10.5 ± 5.9</td>
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<td>-13 ± 78</td>
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<td>-34 ± 40</td>
<td>-30 ± 42</td>
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<td>-9 ± 42</td>
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<td>C-R</td>
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<td>11.3 ± 5.9</td>
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<tr>
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<td>-63 ± 42</td>
<td>-57 ± 42</td>
<td>-41 ± 42</td>
<td>-42 ± 42</td>
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</tbody>
</table>
Conclusions

• **CMB contains lots of useful information!**
  - primordial perturbations + well understood physics (cosmological parameters)

• **Precision cosmology**
  - constrain many cosmological parameters + primordial perturbations

• Currently no evidence for any deviations from standard near scale-invariant purely adiabatic primordial spectrum

• **E-polarization and T-E measure optical depth, constrain reionization; constrain isocurvature modes**

• **Large scale B-mode polarization from primordial gravitational waves:**
  - energy scale of inflation
  - rule out most ekpyrotic and pure curvaton/inhomogeneous reheating models and others

• No evidence for primordial nongaussianity from CMB