Primordial nongaussianities I: cosmic microwave background

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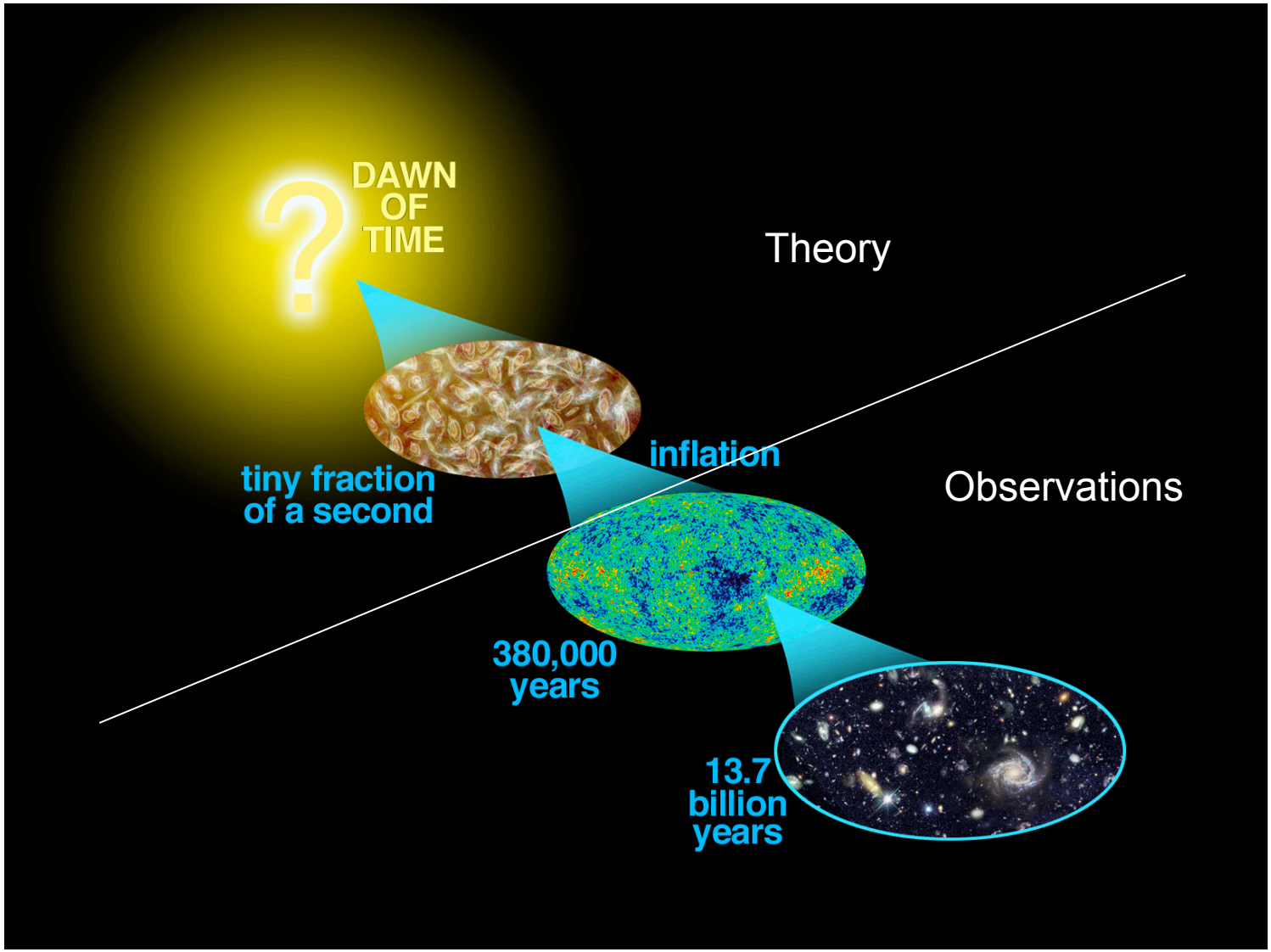
Rio de Janeiro, August 2014

Outline

- Primordial nongaussianity
- Introduction and basic physics
- CMB temperature power spectrum and observables
- Primordial perturbations
- CMB Polarization: E and B modes

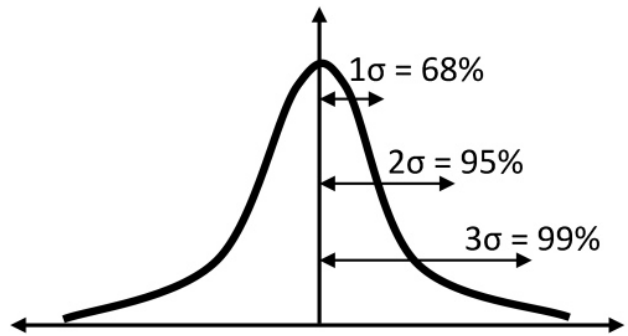


Source: NASA/WMAP Science Team

Gaussian random field, ζ(x)

- normal distribution of values in real space, Prob[ζ(x)]

$$\mathrm{Prob}(\zeta) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\zeta^2 / 2\sigma^2 \right)$$



- defined entirely by power spectrum in Fourier space

$$\left\langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \right\rangle = (2\pi)^3 P_\zeta(k) \delta^3\left(\vec{k} + \vec{k}'\right)$$

- bispectrum and (connected) higher-order correlations vanish

$$\left\langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \zeta_{\vec{k}''} \right\rangle = 0$$

non-Gaussian random field, $\zeta(x)$

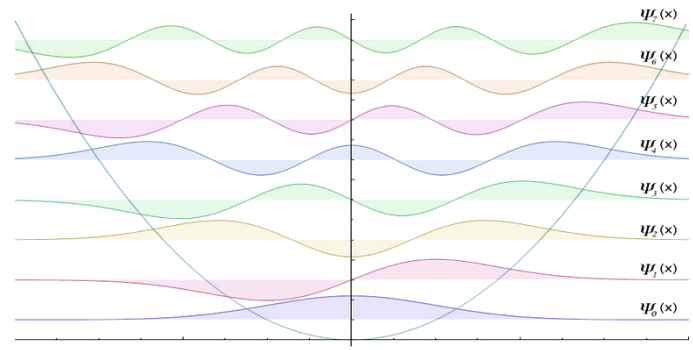
anything else

Rocky Kolb

non-Rocky Kolb

Primordial Gaussianity from inflation

- **Quantum fluctuations from inflation**
  - *ground state of simple harmonic oscillator*
  - *almost free field in almost de Sitter space*
  - *almost scale-invariant and almost Gaussian*



- **Power spectra probe background dynamics (H, ε, ...)**

$$\left\langle \zeta_{k_1} \zeta_{k_2} \right\rangle = (2\pi)^3 P_\zeta(k)\, \delta^3(k_1 + k_2) \quad , \quad P_\zeta(k) \propto k^{n-4}$$

  - *but, many different models, can produce similar power spectra*

- ***Higher-order correlations can distinguish different models***

$$\left\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \right\rangle = (2\pi)^3 B_\zeta(k_1, k_2, k_3)\, \delta^3(k_1 + k_2 + k_3)$$

  - *non-Gaussianity ← non-linearity ← interactions = physics+gravity*

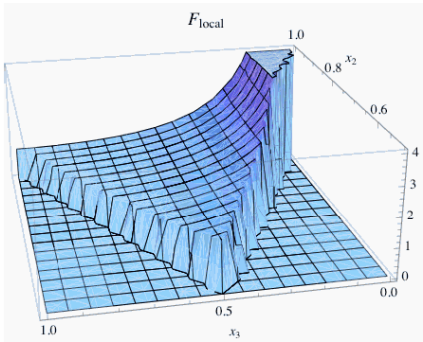
Wikipedia: AllenMcC

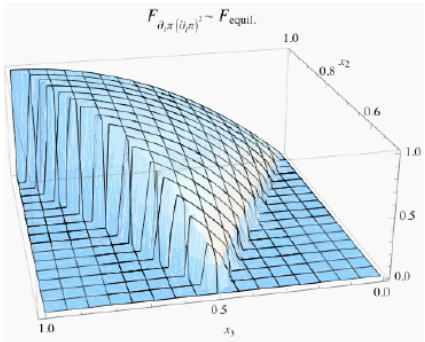
Many sources of non-Gaussianity

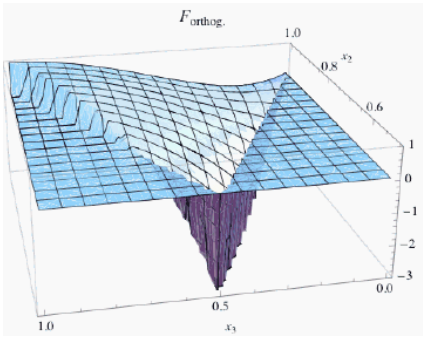
inflation

Initial vacuum	Excited state
Sub-Hubble evolution	Higher-derivative interactions e.g. k-inflation, DBI, Galileons
Hubble-exit	Features in potential
Super-Hubble evolution	Self-interactions + gravity
End of inflation	Tachyonic instability
(p)Reheating	Modulated (p)reheating
After inflation	Curvaton decay
	Magnetic fields
primordial non-Gaussianity	
Primary anisotropies	Last-scattering
Secondary anisotropies	ISW/lensing + foregrounds

Many shapes for primordial bispectra





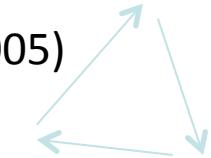


- local type (Komatsu&Spergel 2001)
  - local in real space
  - max for squeezed triangles: k<<k’,k’’

$$B_\xi(k_1,k_2,k_3) \propto \left(\frac{1}{k_1^3k_2^3}+\frac{1}{k_2^3k_3^3}+\frac{1}{k_3^3k_1^3}\right)$$

- equilateral type (Creminelli et al 2005)
  - peaks for k1~k2~k3

$$B_\xi(k_1,k_2,k_3) \propto \left(\frac{3(k_1+k_2-k_3)(k_2+k_3-k_1)(k_3+k_1-k_2)}{k_1^3k_2^3k_3^3}\right)$$

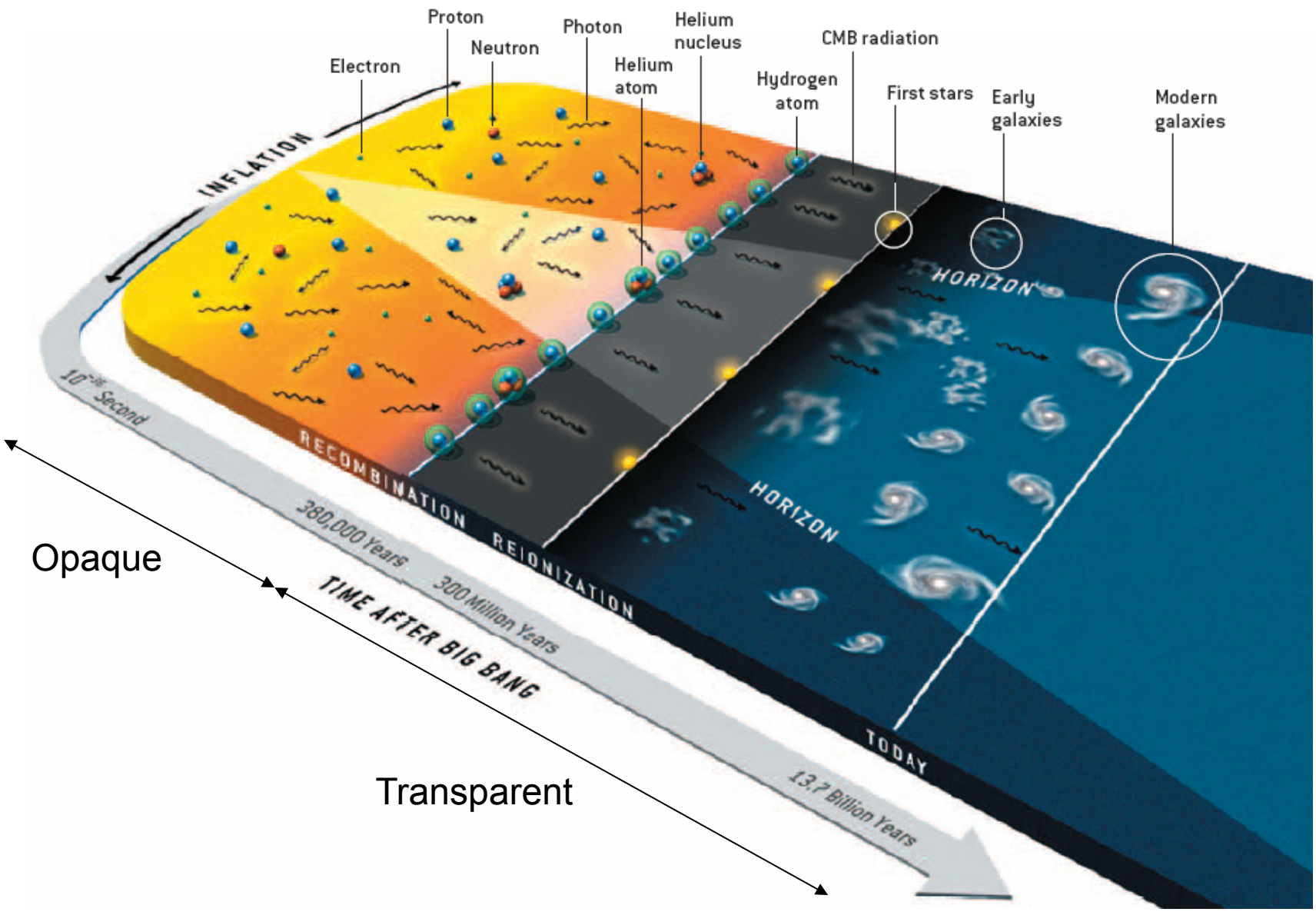


- orthogonal type (Senatore et al 2009)
  - independent of local + equilateral shapes

$$B_\xi(k_1,k_2,k_3) \propto \left(\frac{81}{k_1k_2k_3(k_1+k_2+k_3)^3}\right)$$

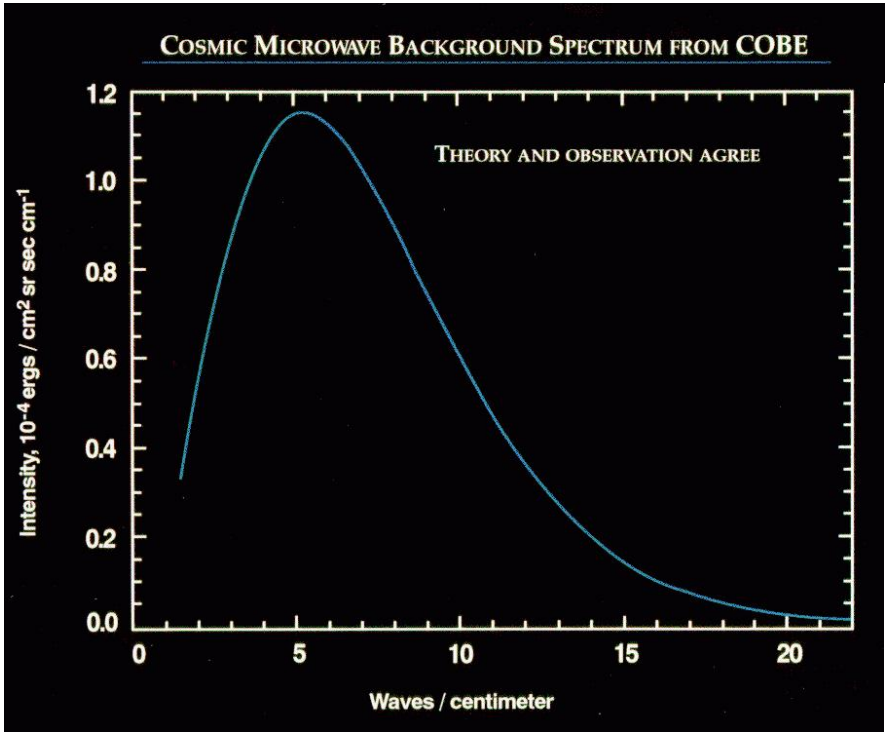
- separable basis (Ferguson et al 2008)

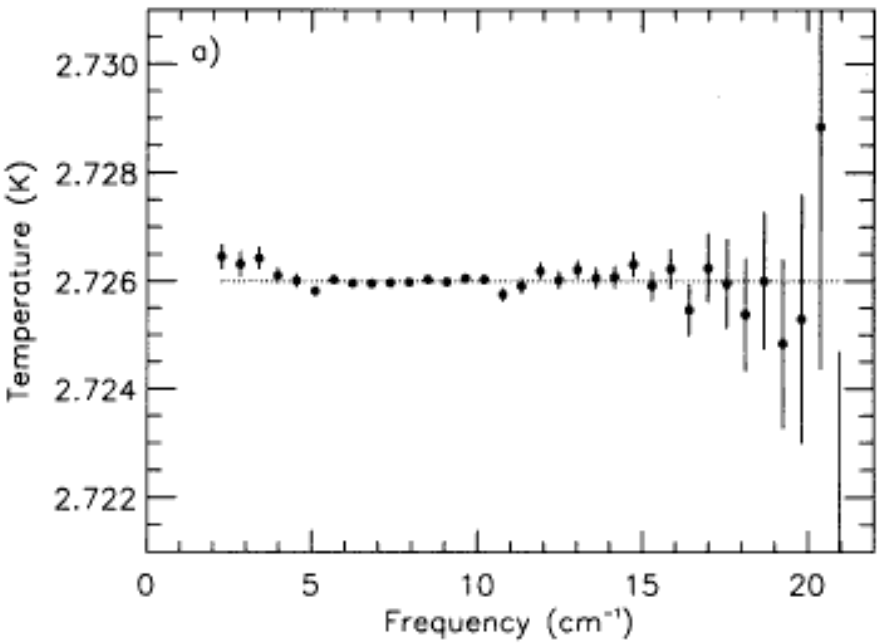
Evolution of the universe



Hu & White, Sci. Am., 290 44 (2004)

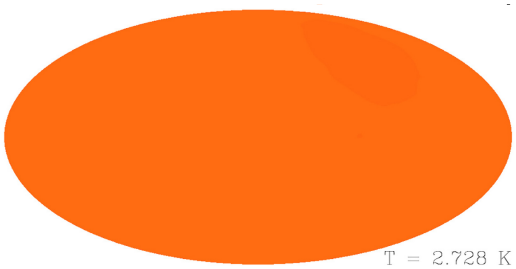
Black body spectrum observed by COBE



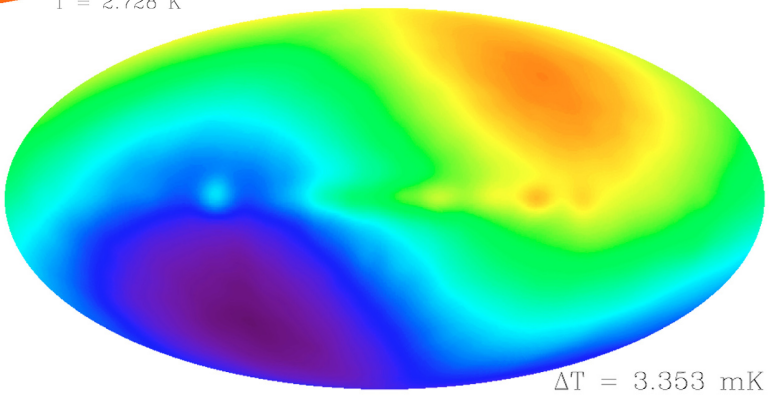


Residuals Mather et al 1994

- close to thermal equilibrium:
  temperature today of 2.726K (~ 3000K at z ~ 1000 because $\nu \sim (1+z)$)

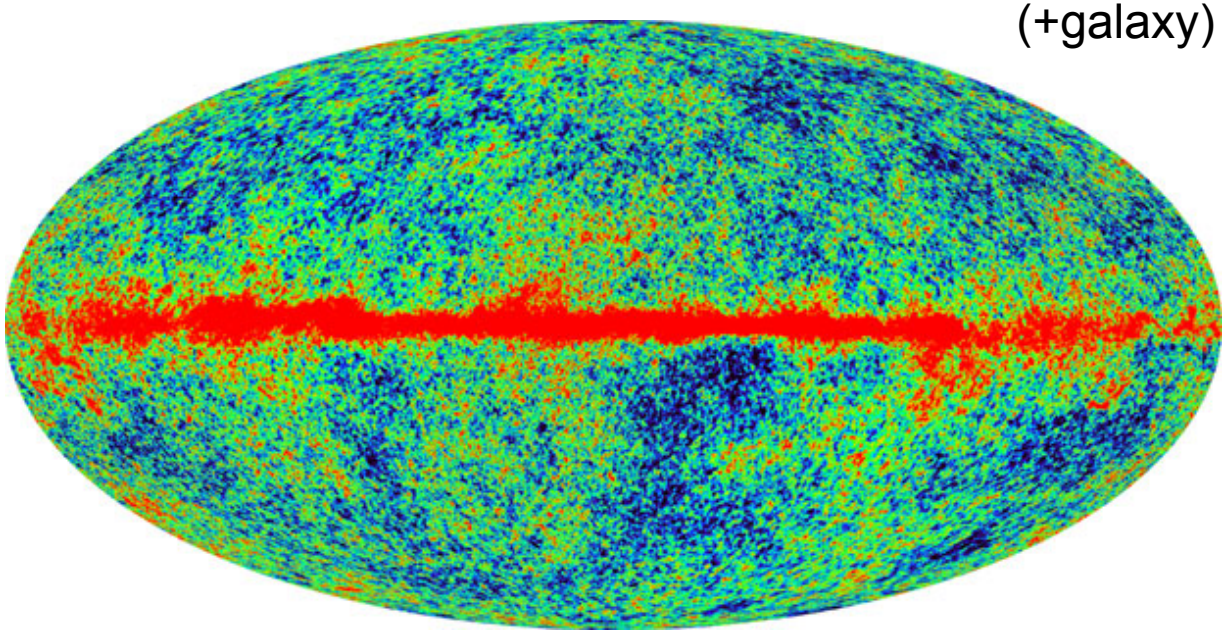


(almost) uniform 2.726K blackbody



Dipole (local motion)

$O(10^{-5})$ perturbations (+galaxy)

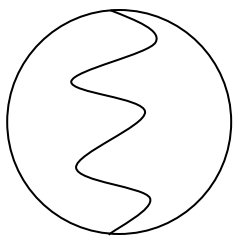


Observations: the microwave sky today

Source: NASA/WMAP Science Team

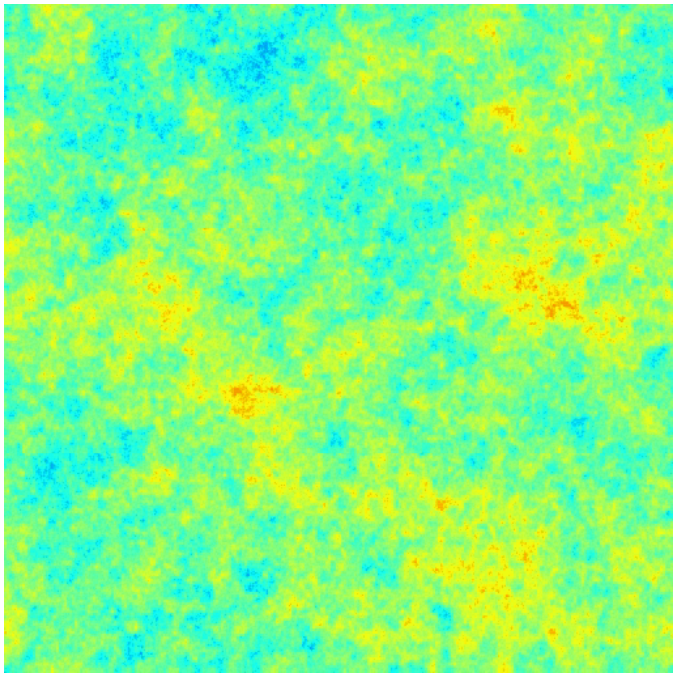
Can we predict the primordial perturbations?

- Maybe..



Quantum Mechanics
"waves in a box" calculation
vacuum state, etc…

Inflation
make >10^{30} times bigger

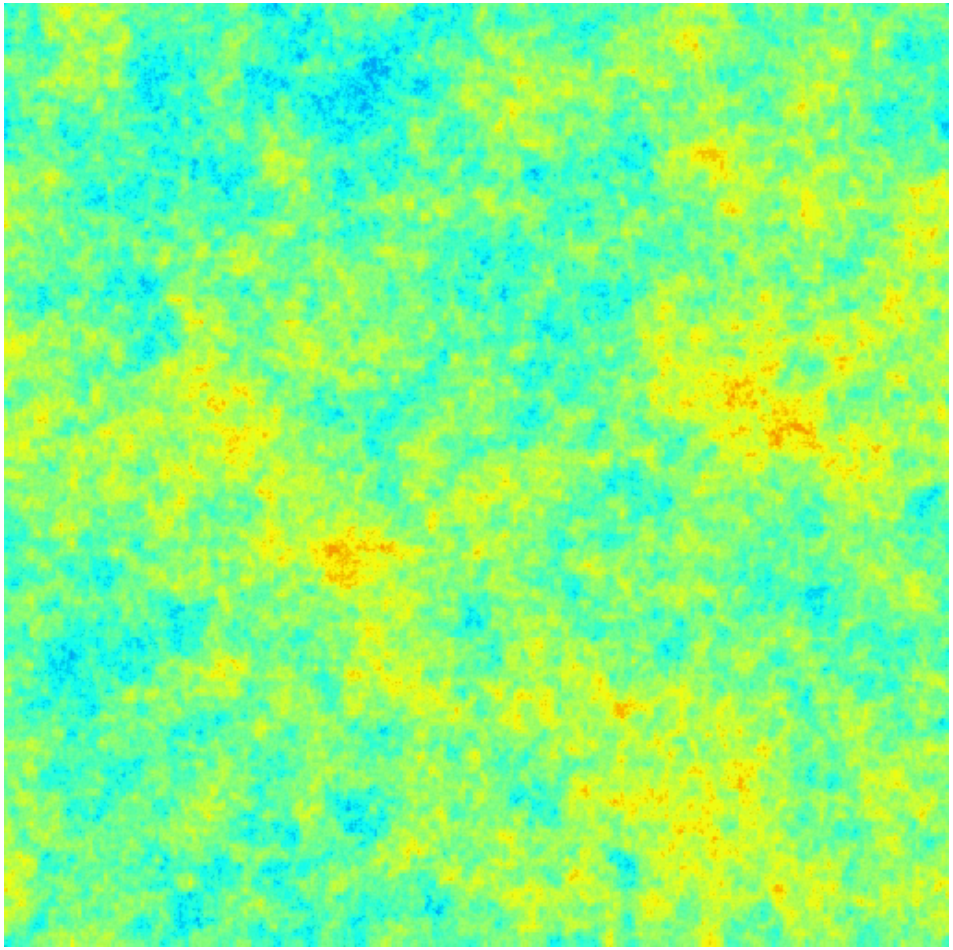


After inflation
Huge size, amplitude ~ 10^{-5}

Perturbation evolution – what we actually observe

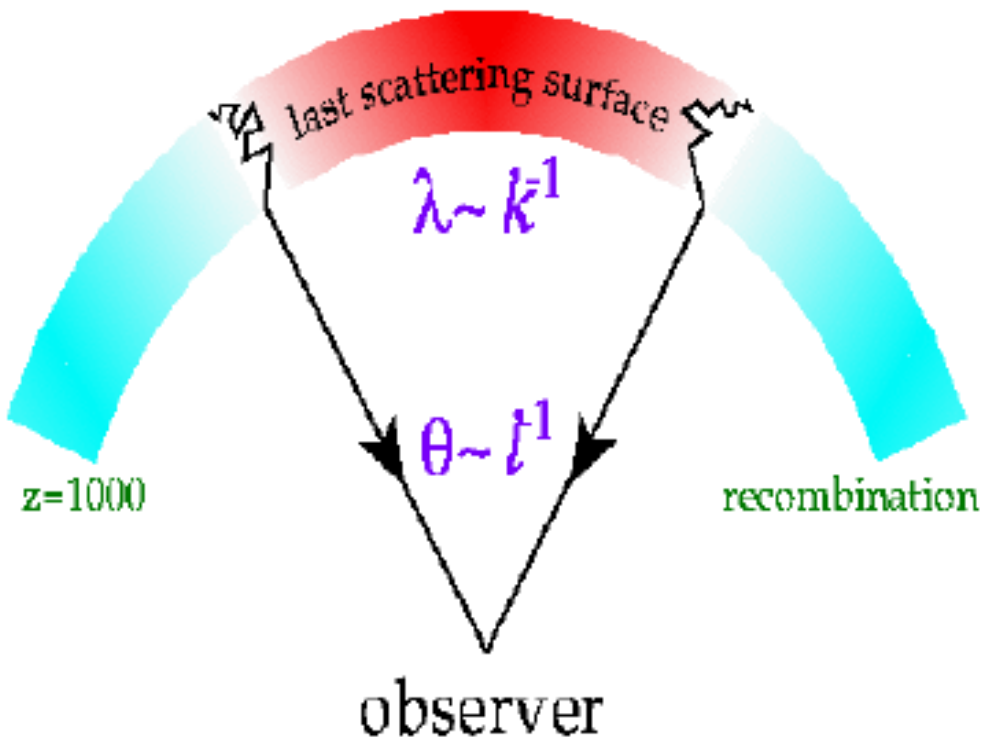
CMB monopole source till 380 000 yrs (last scattering), linear in conformal time
scale invariant primordial adiabatic scalar spectrum

photon/baryon plasma + dark matter, neutrinos



Characteristic scales: sound wave travel distance; diffusion damping length

Observed ΔT
as function of angle on the sky



Calculation of theoretical perturbation evolution

Perturbations $O(10^{-5})$

⇒ Simple linearized equations are very accurate (except small scales)

Can use real or Fourier space

Fourier modes evolve independently: simple to calculate accurately

Physics Ingredients

- Thomson scattering (non-relativistic electron-photon scattering)
  - tightly coupled before recombination: 'tight-coupling' approximation
  (baryons follow electrons because of very strong e-m coupling)
- Background recombination physics (Saha/full multi-level calculation)
- Linearized General Relativity
- Boltzmann equation (how angular distribution function evolves with scattering)

CMB power spectrum C_l

- Theory: Linear physics + Gaussian primordial fluctuations

$$a_{lm} = \int d\Omega \; \Delta T \; Y^*_{lm}$$

Theory prediction $C_l = \langle | a_{lm} |^2 \rangle$

- variance (average over all possible sky realizations)
- statistical isotropy implies independent of *m*

Initial conditions
+ cosmological parameters

$\xrightarrow{\text{linearized GR + Boltzmann equations}}$ C_l

CMBFAST: cmbfast.org
CAMB: camb.info
CMBEASY: cmbeasy.org
COSMICS, etc..

Sources of CMB anisotropy

Sachs Wolfe:
Potential wells at last scattering cause redshifting as photons climb out

Photon density perturbations:
Over-densities of photons look hotter

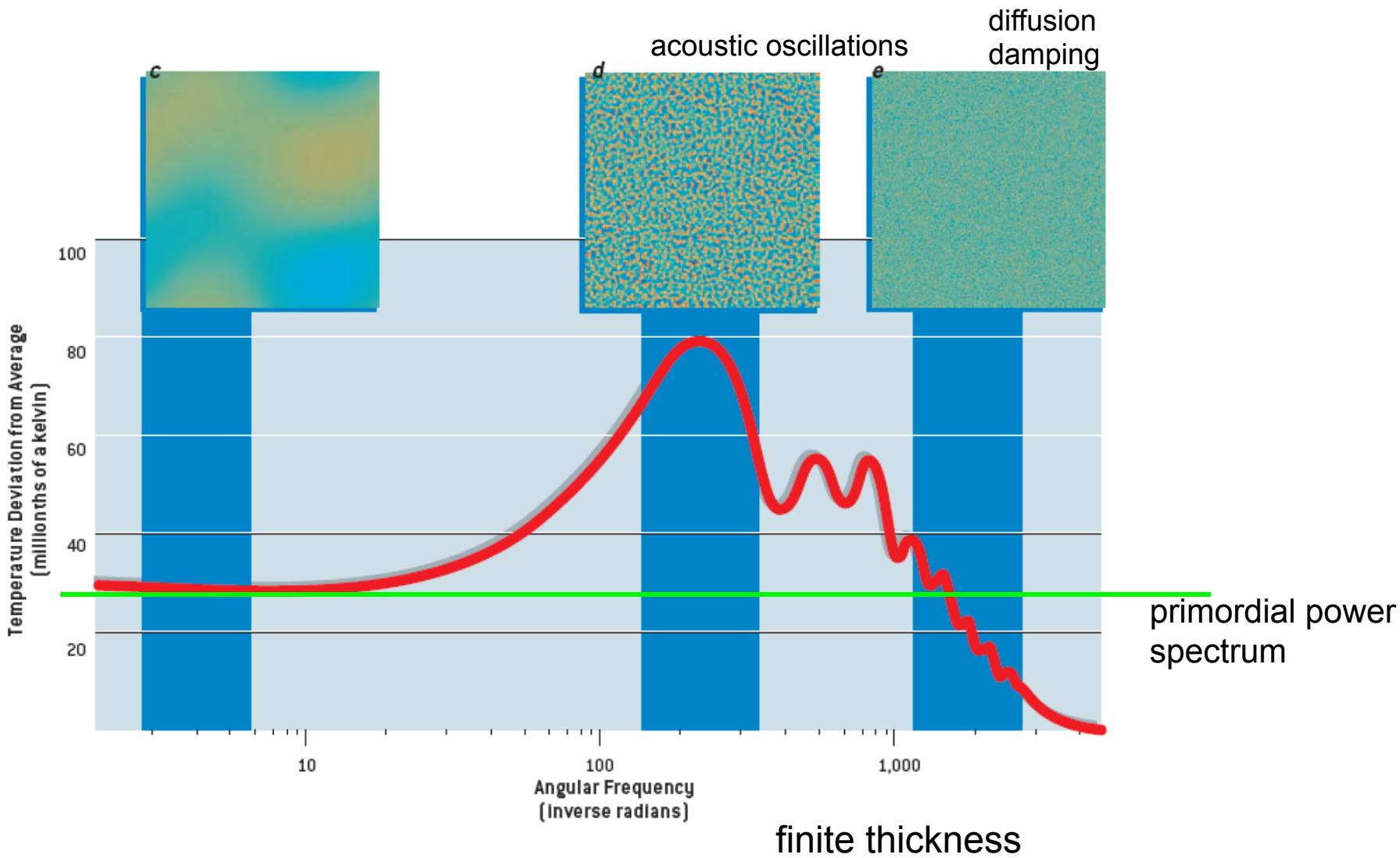
Doppler:
Velocity of photon/baryons at last scattering gives Doppler shift

Integrated Sachs Wolfe:
Evolution of potential along photon line of sight:
net red- or blue-shift as photon climbs in an out of varying potential wells

Others:
Photon quadupole/polarization at last scattering, second-order effects, etc.

CMB temperature power spectrum

Primordial perturbations + later physics



Hu & White, Sci. Am., 290 44 (2004)

Why C_l oscillations?

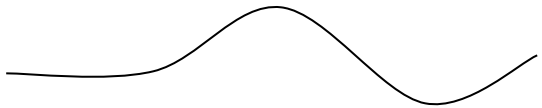
Think in k-space: modes of different size

- Co-moving Poisson equation: $(k/a)^2 \Phi = \kappa \, \delta\rho / 2$
  - potentials approx constant on super-horizon scales
  - radiation domination $\rho \sim 1/a^4$

  → $\delta\rho/\rho \sim k^2 a^2 \Phi$
  → since $\Phi \sim$ constant, super-horizon density perturbations grow $\sim a^2$

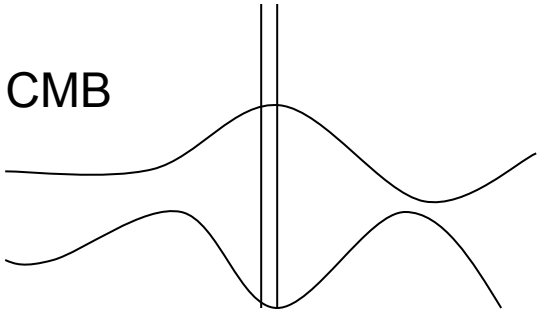
- After entering horizon pressure important: perturbation growth slows, then bounces back

  → series of acoustic oscillations (sound speed $\sim c/\sqrt{3}$)



- CMB anisotropy (mostly) from a surface at fixed redshift: phase of oscillation at time of last scattering depends on time since entering the horizon

  → k-dependent oscillation amplitude in the observed CMB



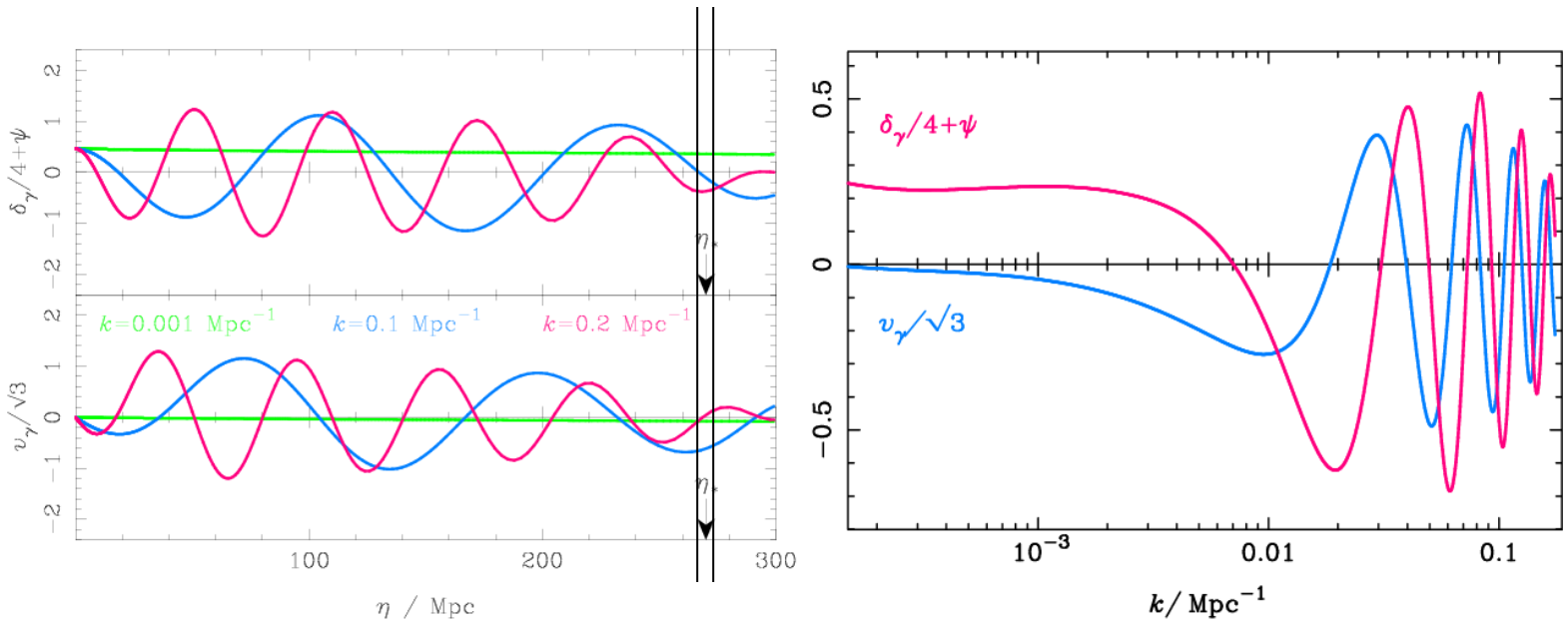
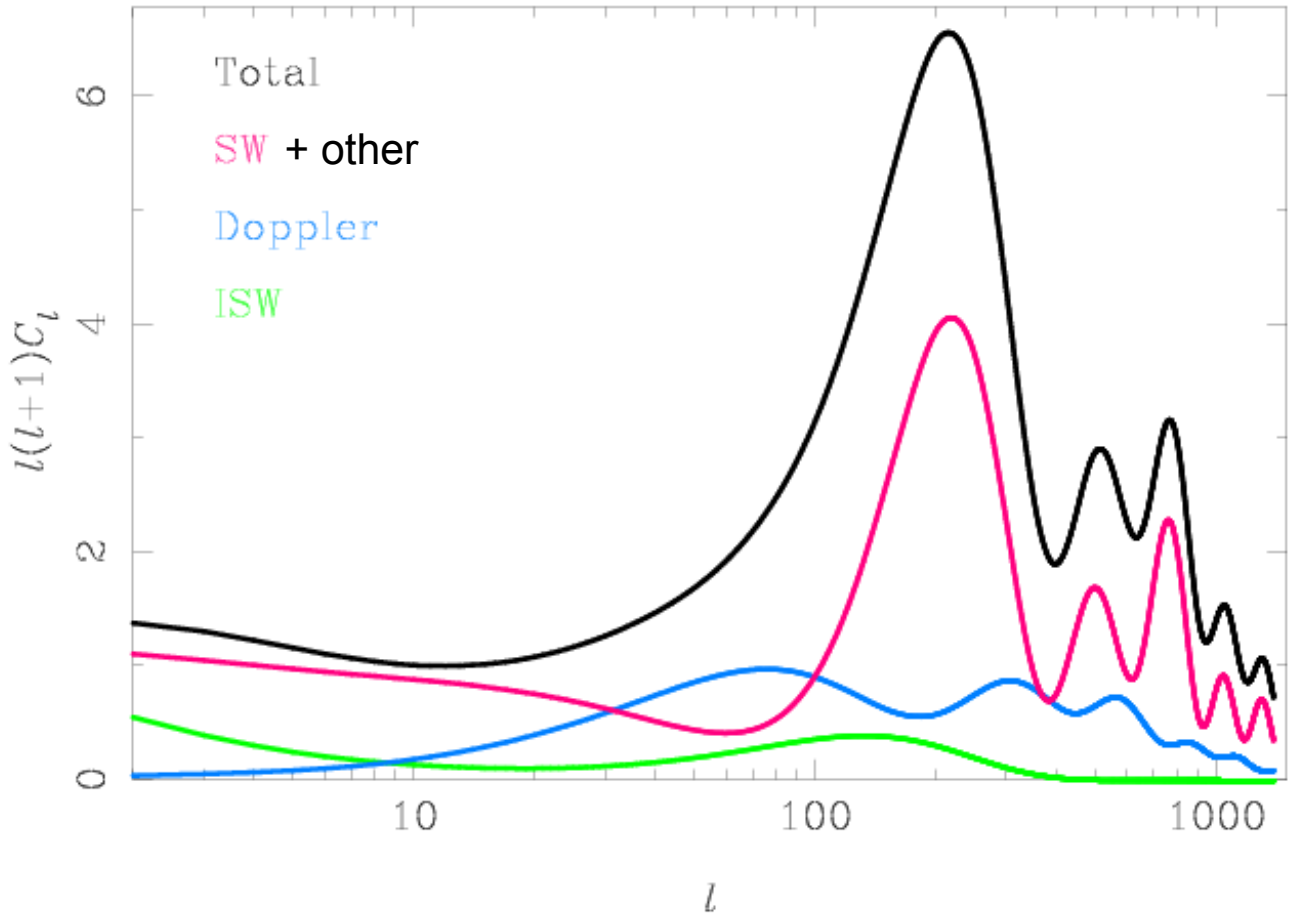


Fig. 3. Evolution of the combination $\delta_\gamma/4 + \psi$ (top left) and the photon velocity v_γ (bottom left) which determine the temperature anisotropies produced at last scattering (denoted by the arrow at η_*). Three modes are shown with wavenumbers $k = 0.001$, 0.1 and $0.2\,\mathrm{Mpc}^{-1}$, and the initial conditions are adiabatic. The fluctuations at the time of last scattering are shown as a function of linear scale in the right-hand plot.

Challinor: astro-ph/0403344

Contributions to temperature C_l



Challinor: astro-ph/0403344

What can we learn from the CMB?

- **Initial conditions**
  What types of perturbations, power spectra, distribution function (Gaussian?);
  => learn about inflation or alternatives.
  (distribution of ΔT; power as function of scale; polarization and correlation)

- **What and how much stuff**
  Matter densities (Ω_b, Ω_{cdm});; neutrino mass
  (details of peak shapes, amount of small scale damping)

- **Geometry and topology**
  global curvature Ω_K of universe; topology
  (angular size of perturbations; repeated patterns in the sky)

- **Evolution**
  Expansion rate as function of time; reionization
  - Hubble constant H_0; dark energy evolution w = pressure/density
  (angular size of perturbations; $l < 50$ large scale power; polarizationr)

- **Astrophysics**
  S-Z effect (clusters), foregrounds, etc.

- Cosmic Variance: only one sky

Use estimator for variance: $$C_l^{obs} = \frac{1}{2l+1}\sum_m |a_{lm}|^2$$

Assume a_{lm} gaussian:

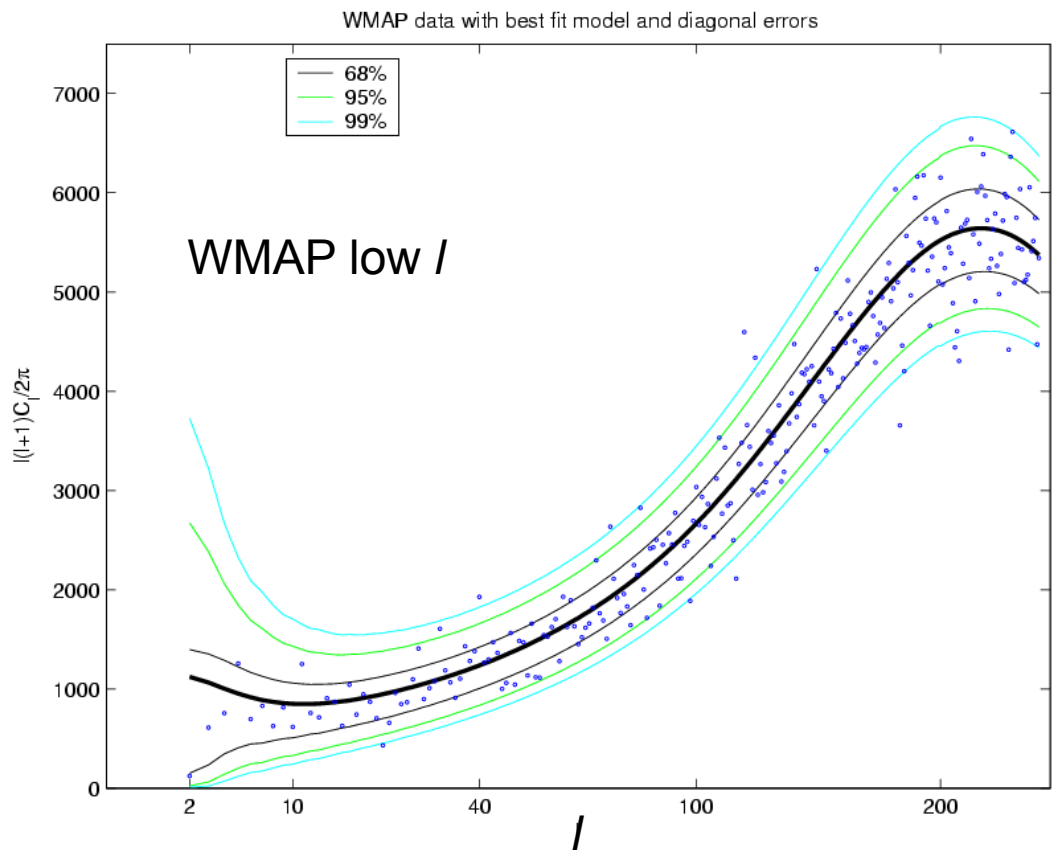
$$C_l^{obs} \sim \chi^2 \text{ with } 2l+1 \text{ d.o.f.}$$

"Cosmic Variance"

$$\left\langle |\Delta C_l^{obs}|^2 \right\rangle \approx \frac{2C_l^2}{2l+1}$$

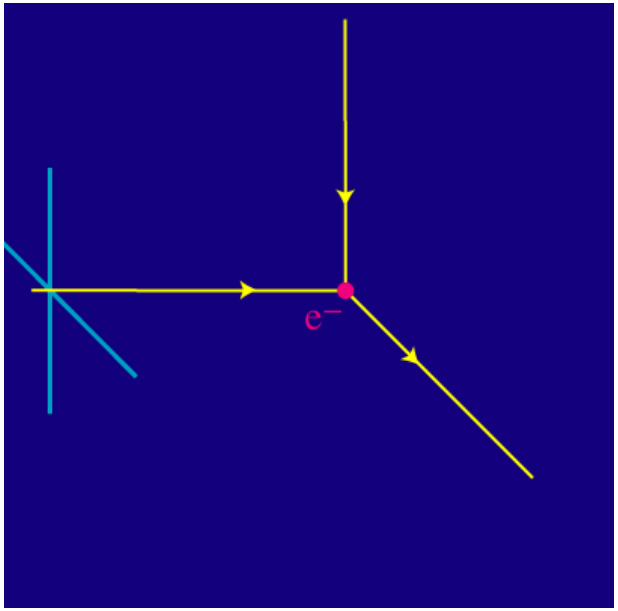
$$P(C_l \mid C_l^{obs})$$

- inverse gamma distribution
(+ noise, sky cut, etc).



Cosmic variance gives fundamental limit on how much we can learn from CMB

How to generate CMB polarization?



(Wayne Hu, CMB Tutorials)

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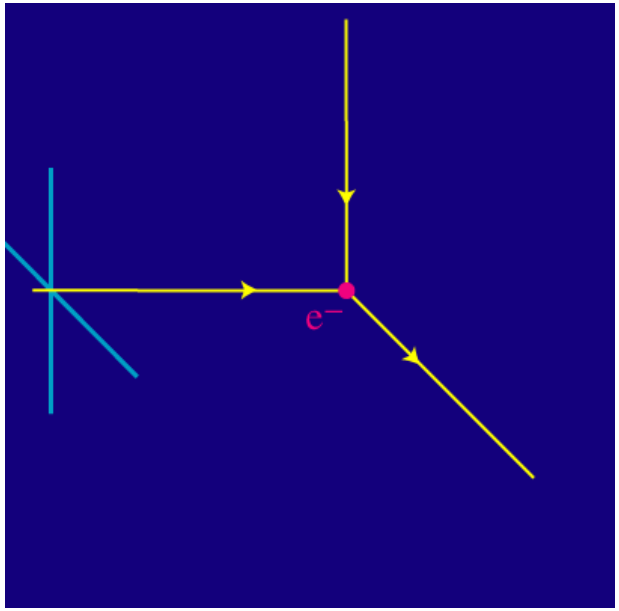

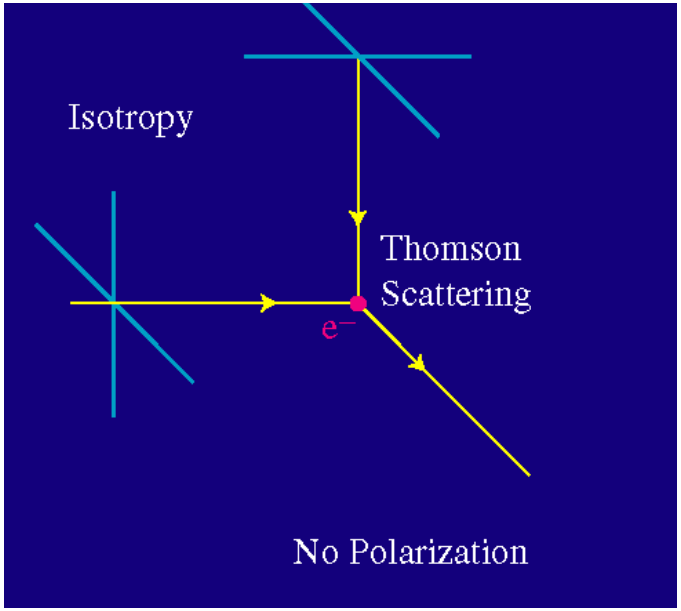

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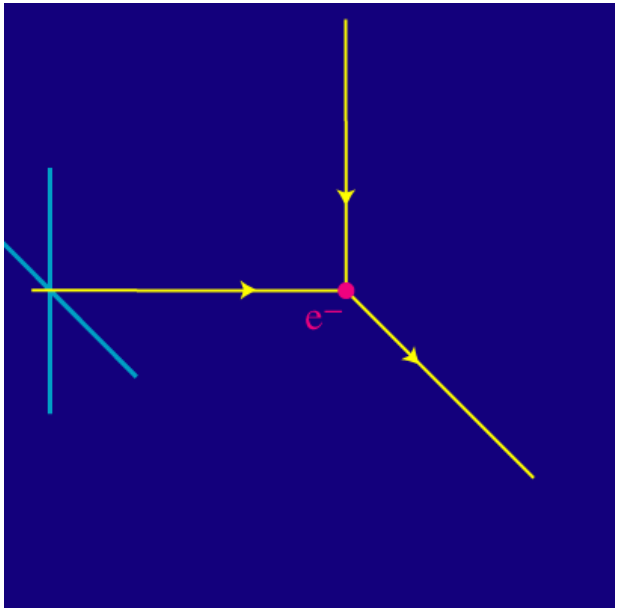

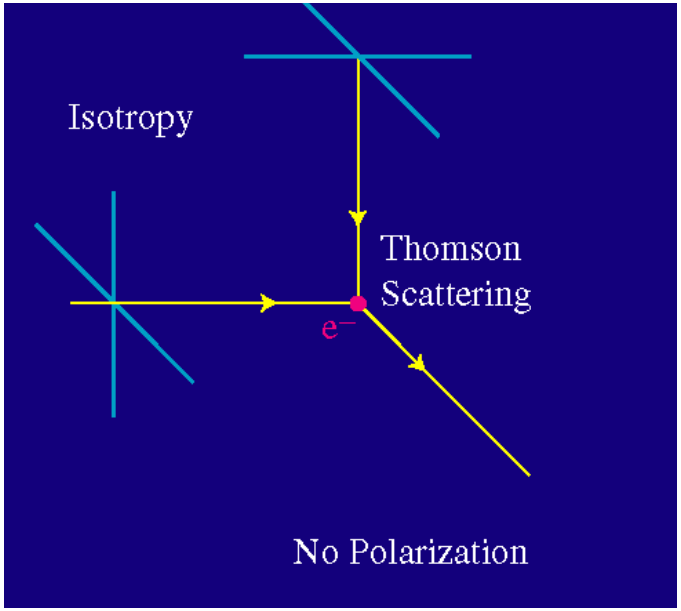

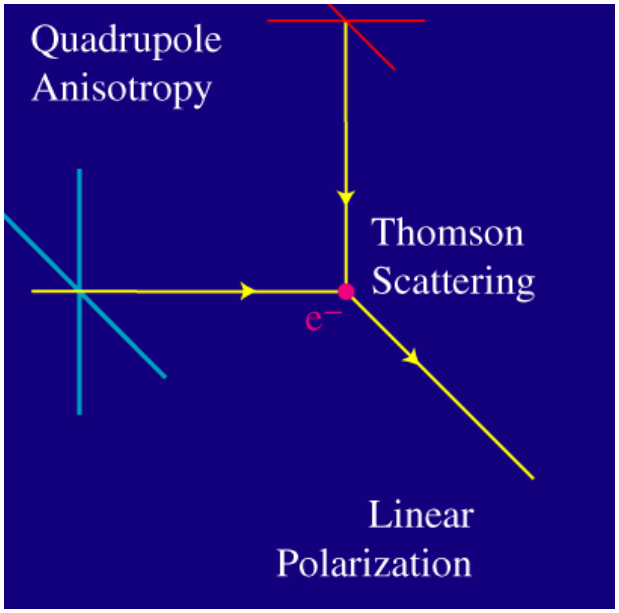

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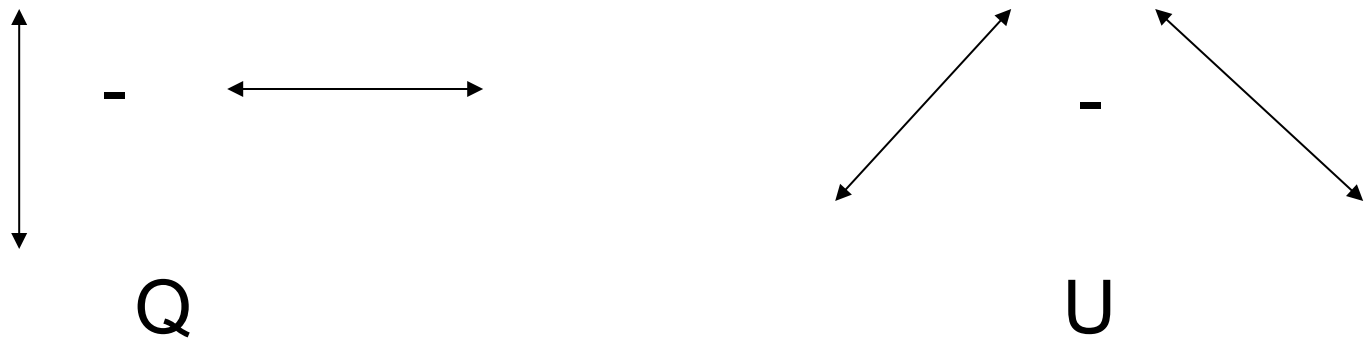
Polarization: Stokes' Parameters

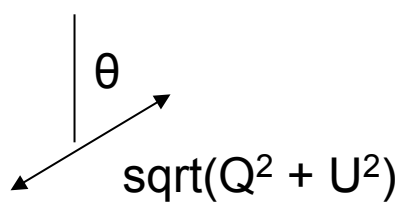


$Q \rightarrow -Q$, $U \rightarrow -U$ under 90 degree rotation

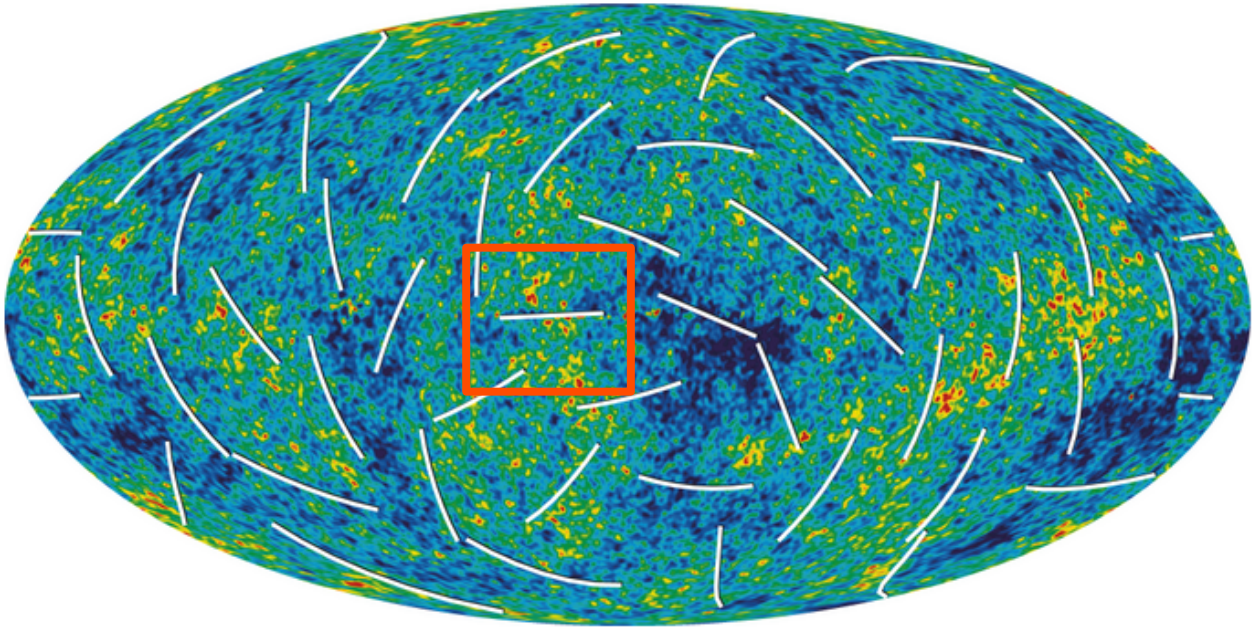
$Q \rightarrow U$, $U \rightarrow -Q$ under 45 degree rotation

Spin-2 field Q + i U
or Rank 2 trace free symmetric tensor

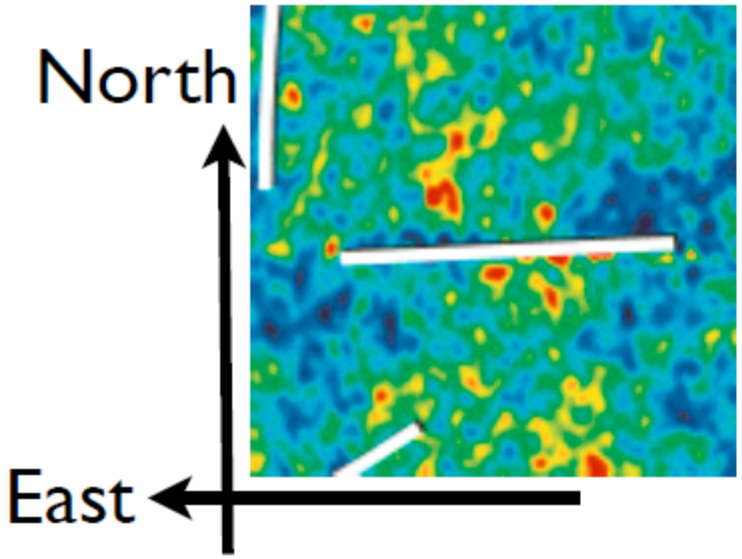
$$P = \begin{pmatrix} Q & U \\ U & -Q \end{pmatrix}$$

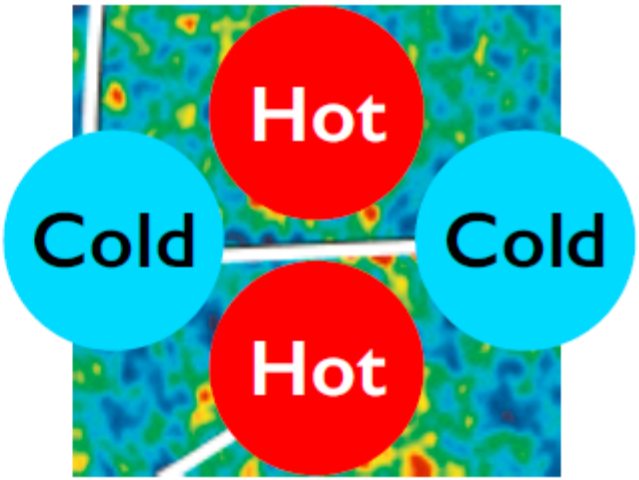


$\theta = \frac{1}{2} \tan^{-1} U/Q$



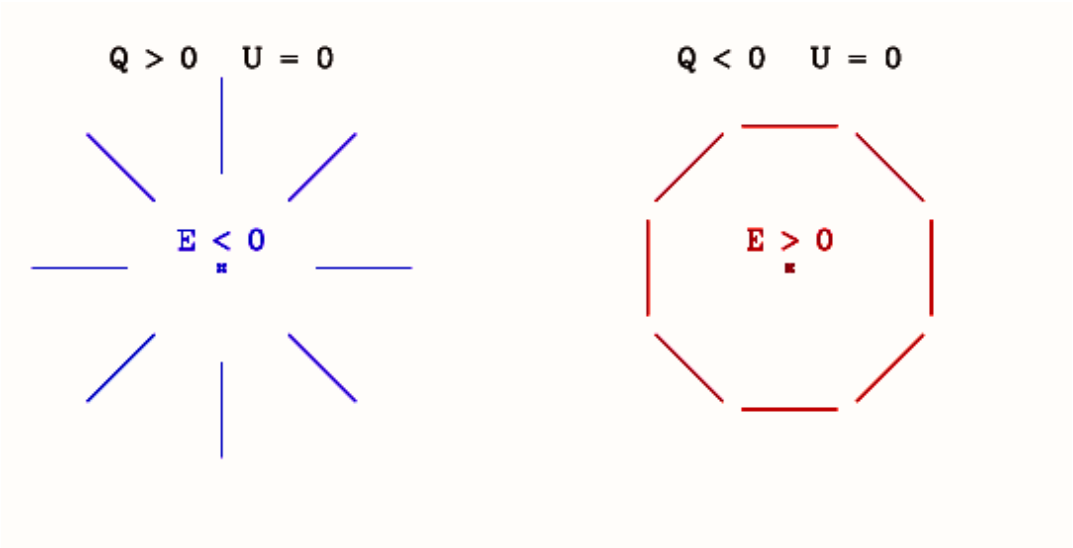
NASA/WMAP Science Team





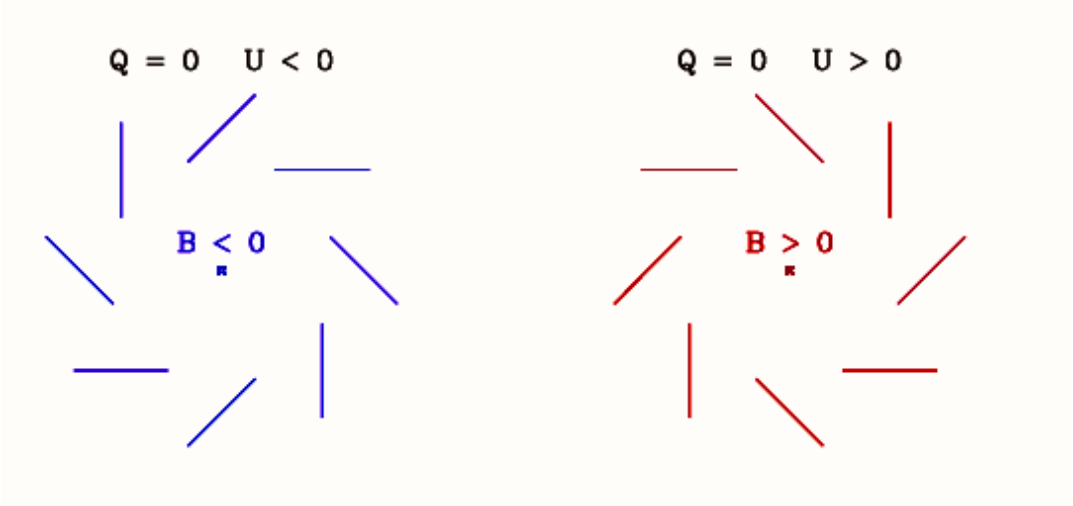
E and B mode patterns

E-mode



Unchanged if seen through the mirror

B-mode



Pattern reverses if seen through the mirror

Seljak and Zaldarriaga, 1998

E and B harmonics

- Expand scalar P_E and P_B in spherical harmonics
- Expand P_{ab} in tensor spherical harmonics

$$\mathcal{P}_{ab} = \frac{1}{\sqrt{2}} \sum_{lm} \left(E_{lm}\, Y^{G}_{(lm)ab} + B_{lm}\, Y^{C}_{(lm)ab} \right)$$

$$E_{lm} = \sqrt{2} \int_{4\pi} \mathrm{d}S\, Y^{G\,ab*}_{(lm)} \mathcal{P}_{ab} \qquad B_{lm} = \sqrt{2} \int_{4\pi} \mathrm{d}S\, Y^{C\,ab*}_{(lm)} \mathcal{P}_{ab}$$

Harmonics are orthogonal over the full sky:

<u>E/B decomposition is exact and lossless on the full sky</u>

Zaldarriaga, Seljak: astro-ph/9609170
Kamionkowski, Kosowsky, Stebbins: astro-ph/9611125

CMB Polarization Signals

- E polarization from scalar, vector and tensor modes
- B polarization only from vector and tensor modes (curl grad = 0) + non-linear scalars

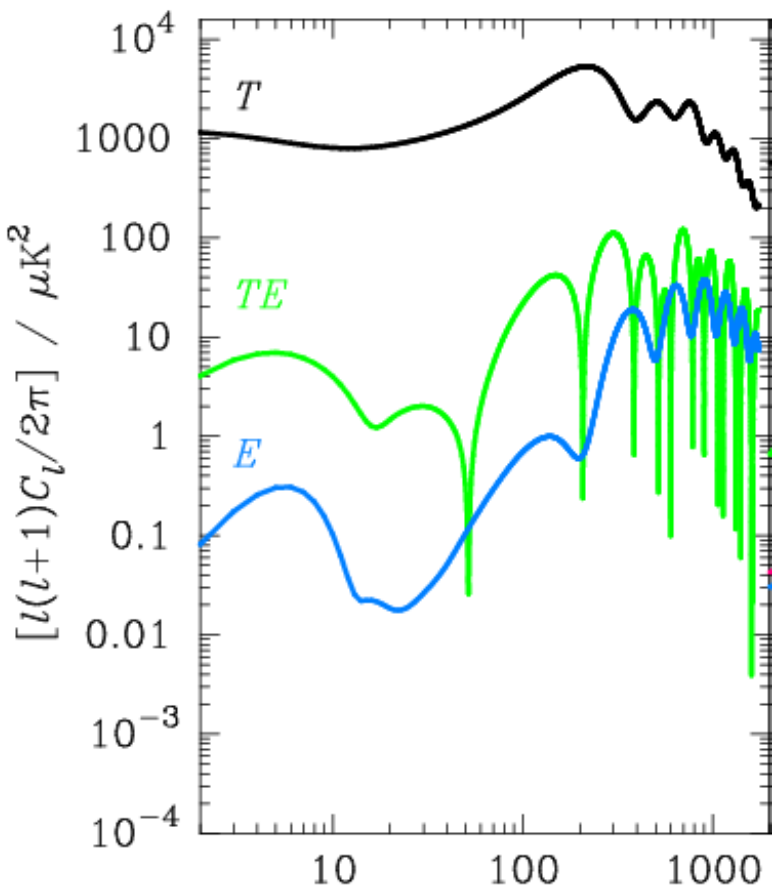
Average over possible realizations (statistically isotropic):

$$\langle E^*_{l'm'} E_{lm} \rangle = \delta_{l'l} \delta_{m'm} C_l^{EE} \qquad \langle B^*_{l'm'} B_{lm} \rangle = \delta_{l'l} \delta_{m'm} C_l^{BB}$$

Parity symmetric ensemble: $\langle E^*_{l'm'} B_{lm} \rangle = 0$

Power spectra contain all the useful information if the field is Gaussian

Scalar adiabatic mode



E polarization only

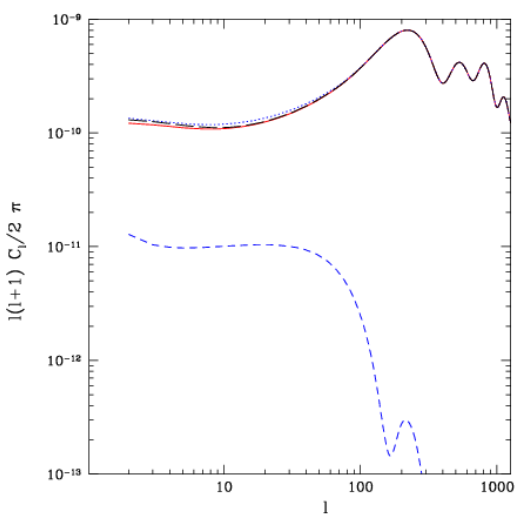
correlation to temperature T-E

Primordial Gravitational Waves
(tensor modes)

- Well motivated by some inflationary models
  - Amplitude measures inflaton potential at horizon crossing
  - distinguish models of inflation

- Observation would rule out other models
  - ekpyrotic scenario predicts exponentially small amplitude
  - small also in many models of inflation, esp. two field e.g. curvaton

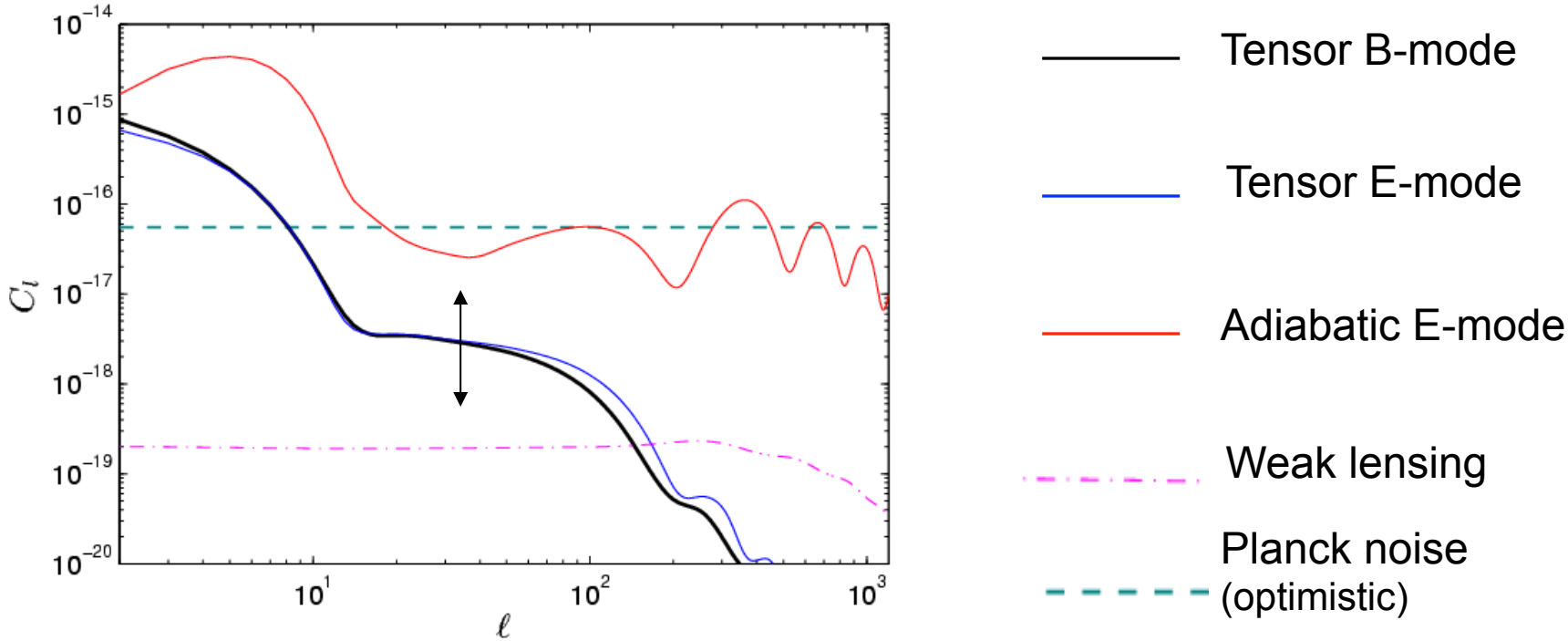
- Weakly constrained from CMB temperature anisotropy
  - cosmic variance limited to 10%
  - degenerate with other parameters (tilt, reionization, etc)



Look at CMB polarization:
'B-mode' smoking gun

Has BICEP2 observed it?

CMB polarization from primordial gravitational waves (tensors)



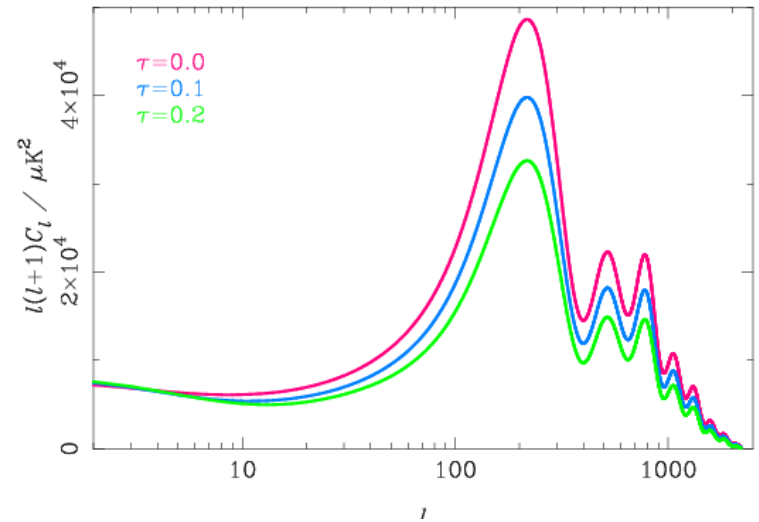
- Amplitude of tensors unknown
- Clear signal from B modes – there are none from scalar modes
- Tensor B is always small compared to adiabatic E

Seljak, Zaldarriaga: astro-ph/9609169

Reionization

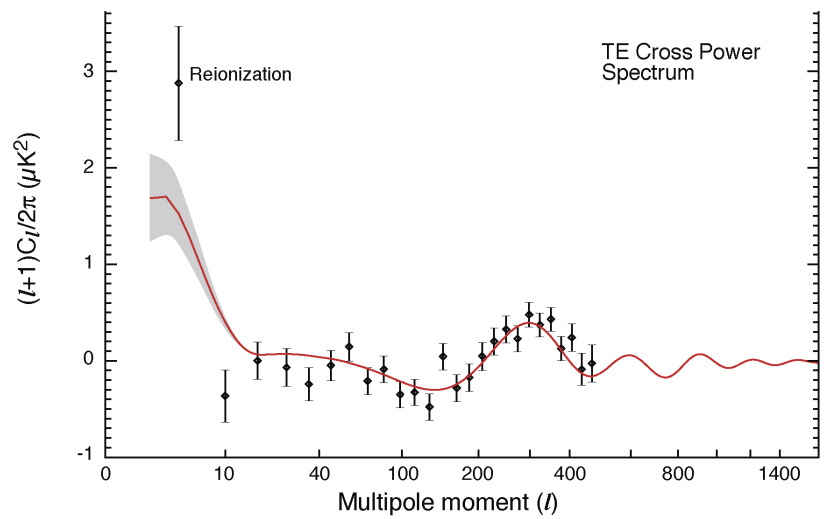
Ionization since z ~ 6-20 scatters CMB photons

Temperature signal similar to tensors



Quadrupole at reionization implies large scale polarization signal

<u>Measure optical depth with WMAP T-E correlation</u>

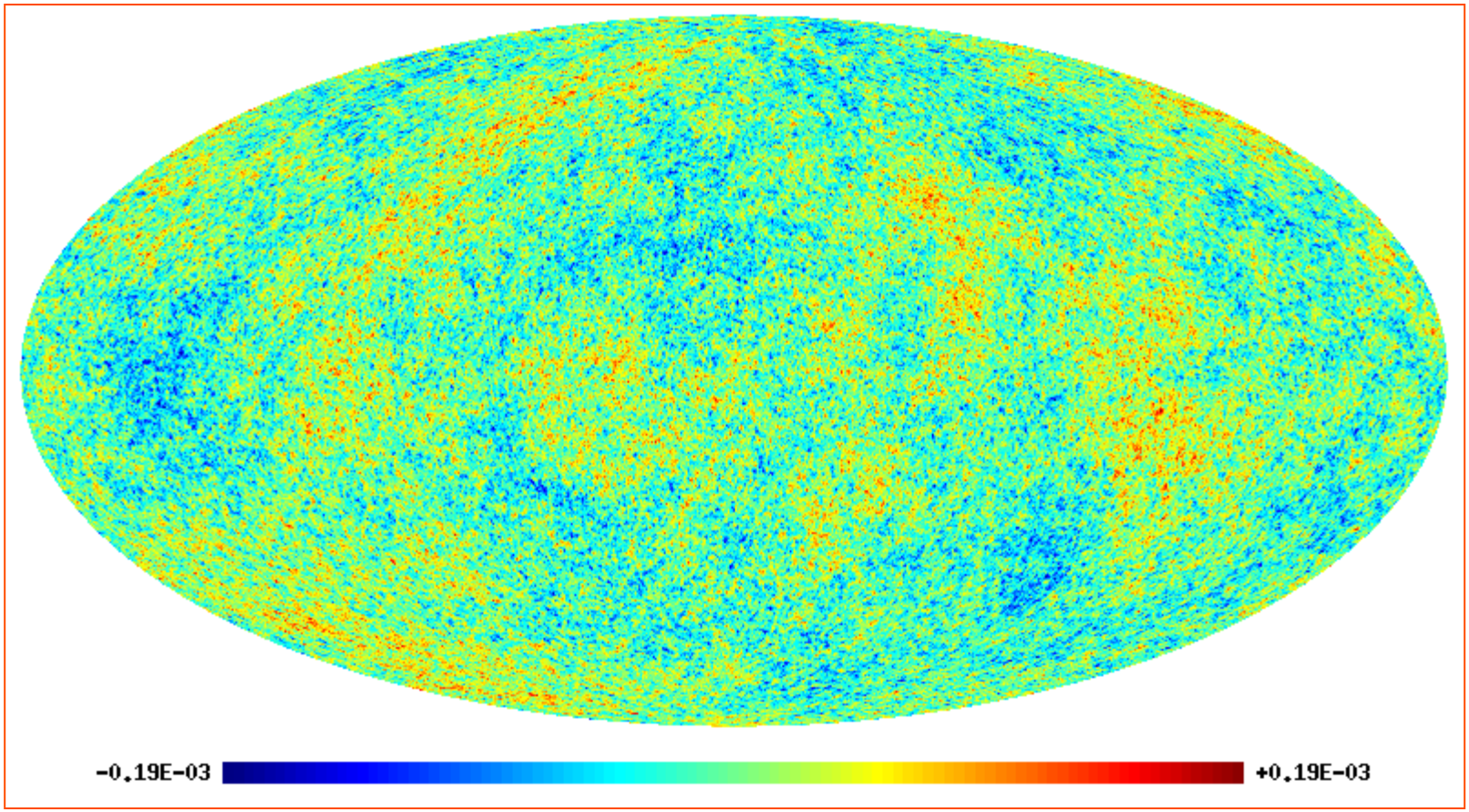


Other non-linear effects

- Thermal Sunyaev-Zeldovich
  Inverse Compton scattering from hot gas: frequency dependent signal
- Kinetic Sunyaev-Zeldovich (kSZ)
  Doppler from bulk motion of clusters; patchy reionization; (almost) frequency independent signal
- Ostriker-Vishniac (OV)
  same as kSZ but for early linear bulk motion
- Rees-Sciama
  Integrated Sachs-Wolfe from evolving non-linear potentials: frequency independent
- Gravitational lensing
  discussed on friday

Newtonian potential a ***Gaussian random field***

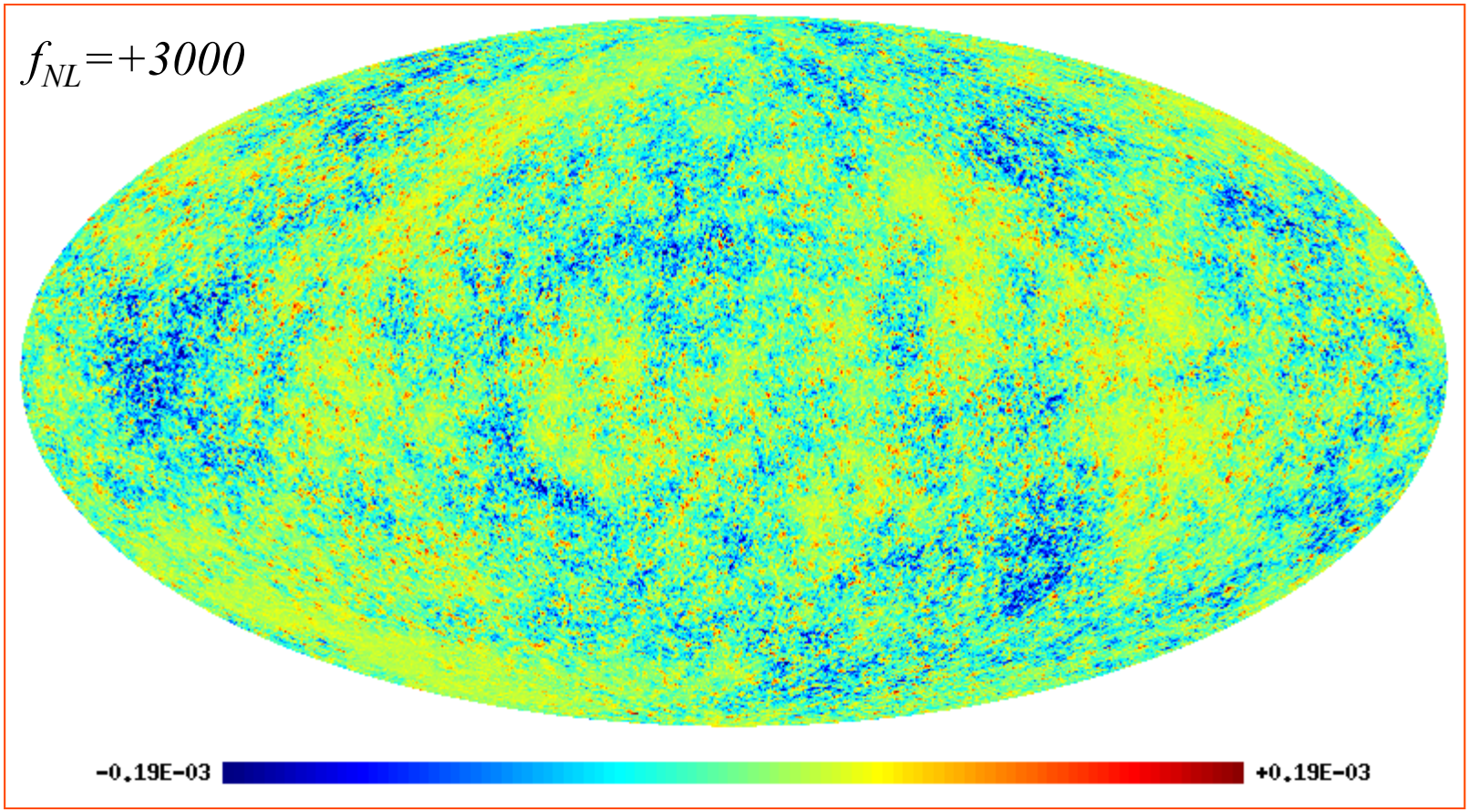
$\Phi(x) = \phi_G(x)$



Liguori, Matarrese and Moscardini (2003)

Newtonian potential ***a local function of Gaussian random field***

$$\Phi(x) = \phi_G(x) + f_{NL}\,(\,\phi_G^2(x) - \langle\phi_G^2\rangle\,)$$



$\Delta T/T \approx -\Phi/3$, so positive f_{NL} $\Rightarrow$ more cold spots in CMB

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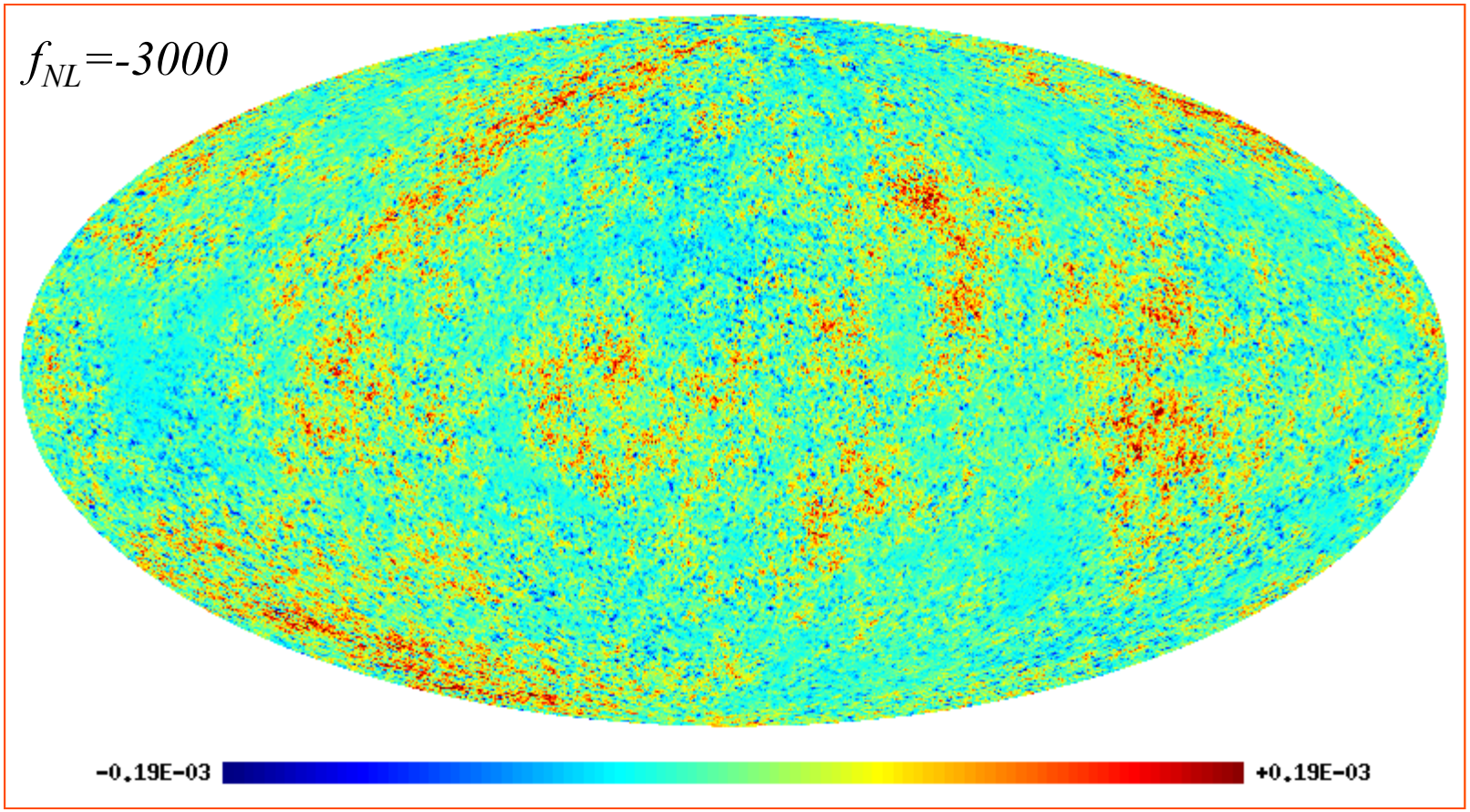


$\Delta T/T \approx -\Phi/3$, so negative f_{NL} $\Rightarrow$ more hot spots in CMB

Liguori, Matarrese and Moscardini (2003)

Newtonian potential a local function of ***Gaussian random field***

$$\Phi(x) = \phi_G(x) + f_{NL} \left(\phi_G^2(x) - \langle \phi_G^2 \rangle \right)$$

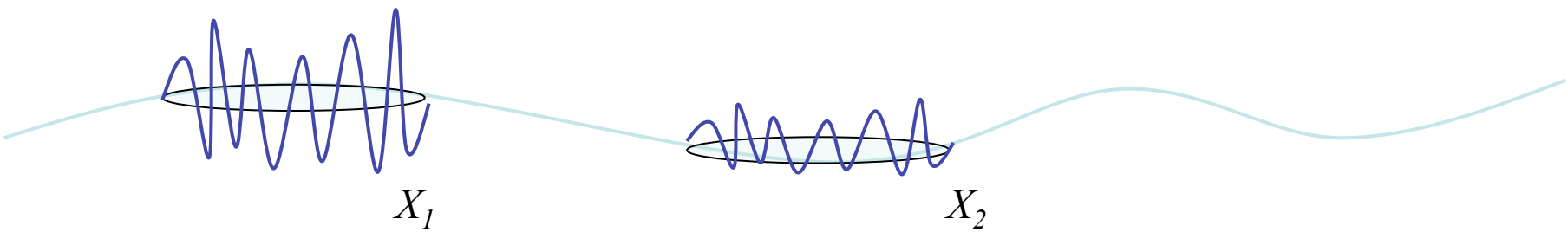
⇒ ***Large-scale modulation of small-scale power***

split Gaussian field into long (L) and short (s) wavelengths

$$\phi_G (X+x) = \phi_L(X) + \phi_s(x)$$

two-point function on small scales for given ϕ_L

$$\langle \Phi(x_1)\, \Phi(x_2) \rangle_L = (1+4 f_{NL}\, \phi_L) \langle \phi_s (x_1)\, \phi_s (x_2) \rangle + \ldots$$



i.e., ***inhomogeneous modulation of small-scale power***

$$P(k, X) \rightarrow [\, 1 + 4 f_{NL}\, \phi_L(X) \,]\, P_s(k)$$

but $f_{NL} < 5$ *so any effect must be* 10^{-4}

Optimal estimation from CMB data

$$\hat{B}_{\ell_1\ell_2\ell_3} = \int d^2\hat{n} T_{\ell_1}(\hat{n}) T_{\ell_2}(\hat{n}) T_{\ell_3}(\hat{n}) \,,$$

where the filtered maps $T_\ell(\hat{n})$ are defined as:

$$T_\ell(\hat{n}) \equiv \sum_m a_{\ell m} Y_{\ell m}(\hat{n}) \,.$$

$$\hat{f}_{\rm NL} = \frac{6}{N} \sum_{\ell_1 \le \ell_2 \le \ell_3} \frac{B^{\rm th}_{\ell_1\ell_2\ell_3}\left(B^{\rm obs}_{\ell_1\ell_2\ell_3} - B^{\rm lin}_{\ell_1\ell_2\ell_3}\right)}{V_{\ell_1\ell_2\ell_3}} .$$

$$V_{\ell_1\ell_2\ell_3} = g_{\ell_1\ell_2\ell_3} h^2_{\ell_1\ell_2\ell_3} C_{\ell_1} C_{\ell_2} C_{\ell_3}$$

$$h_{\ell_1\ell_2\ell_3} = \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix}$$

Planck Paper XXIV constraints: no evidence for primordial nongaussianity

	f_{NL}					
	Independent			ISW-lensing subtracted		
Shape	KSW	Binned	Modal	KSW	Binned	Modal
SMICA						
Local	9.8 ± 5.8	9.2 ± 5.9	8.3 ± 5.9	**2.7 ± 5.8**	2.2 ± 5.9	1.6 ± 6.0
Equilateral	−37 ± 75	−20 ± 73	−20 ± 77	**−42 ± 75**	−25 ± 73	−20 ± 77
Orthogonal	−46 ± 39	−39 ± 41	−36 ± 41	**−25 ± 39**	−17 ± 41	−14 ± 42
NILC						
Local	11.6 ± 5.8	10.5 ± 5.8	9.4 ± 5.9	4.5 ± 5.8	3.6 ± 5.8	2.7 ± 6.0
Equilateral	−41 ± 76	−31 ± 73	−20 ± 76	−48 ± 76	−38 ± 73	−20 ± 78
Orthogonal	−74 ± 40	−62 ± 41	−60 ± 40	−53 ± 40	−41 ± 41	−37 ± 43
SEVEM						
Local	10.5 ± 5.9	10.1 ± 6.2	9.4 ± 6.0	3.4 ± 5.9	3.2 ± 6.2	2.6 ± 6.0
Equilateral	−32 ± 76	−21 ± 73	−13 ± 77	−36 ± 76	−25 ± 73	−13 ± 78
Orthogonal	−34 ± 40	−30 ± 42	−24 ± 42	−14 ± 40	−9 ± 42	−2 ± 42
C-R						
Local	12.4 ± 6.0	11.3 ± 5.9	10.9 ± 5.9	6.4 ± 6.0	5.5 ± 5.9	5.1 ± 5.9
Equilateral	−60 ± 79	−52 ± 74	−33 ± 78	−62 ± 79	−55 ± 74	−32 ± 78
Orthogonal	−76 ± 42	−60 ± 42	−63 ± 42	−57 ± 42	−41 ± 42	−42 ± 42

Conclusions

- CMB contains lots of useful information!
  - primordial perturbations + well understood physics (cosmological parameters)
- Precision cosmology
  - constrain many cosmological parameters + primordial perturbations
- Currently no evidence for any deviations from standard near scale-invariant purely adiabatic primordial spectrum
- E-polarization and T-E measure optical depth, constrain reionization; constrain isocurvature modes
- Large scale B-mode polarization from primordial gravitational waves:
  - energy scale of inflation
  - rule out most ekpyrotic and pure curvaton/ inhomogeneous reheating models and others
- No evidence for primordial nongaussianity from CMB